ANALYTICAL AND FEM MODELING OF ALUMINUM BILLET INDUCTION HEATING WITH EXPERIMENTAL VERIFICATION

NTNU
Innovation and Creativity

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TMS2012 141st Annual Meeting & Exhibition

March 11-15 | Orlando, Florida

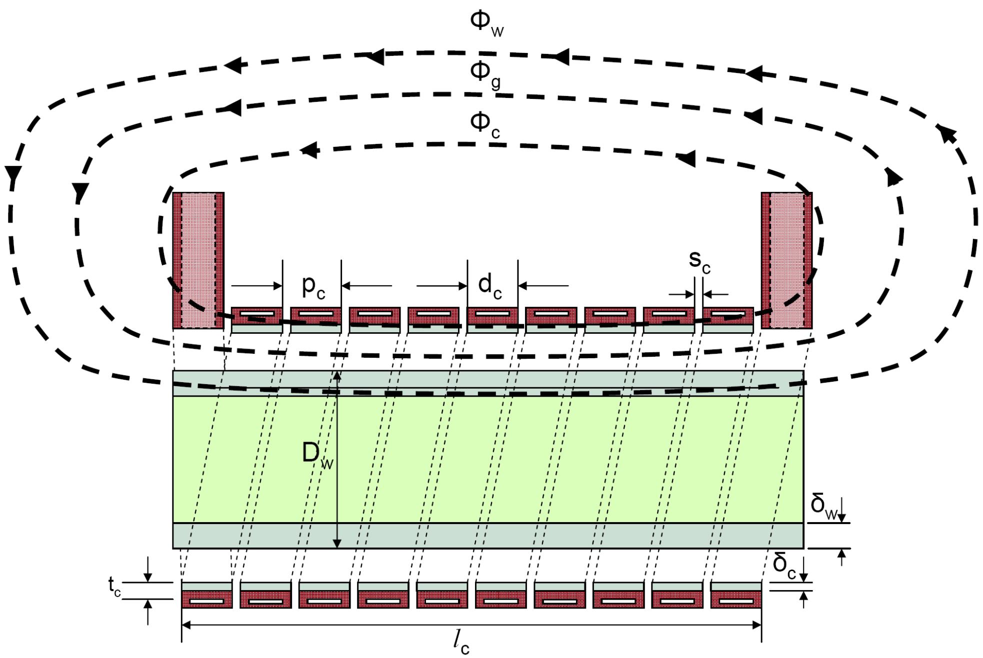
Heating will vary by the square of the flux density in the air-gap.

$$P_w = \frac{\sqrt{2}\pi l}{\mu^2}\left(\boxed{\frac{k_N^* \mu I_c N_c}{l_c}}\right)^2 \frac{1}{\sigma_w}\xi_w \varphi(\xi_w)$$

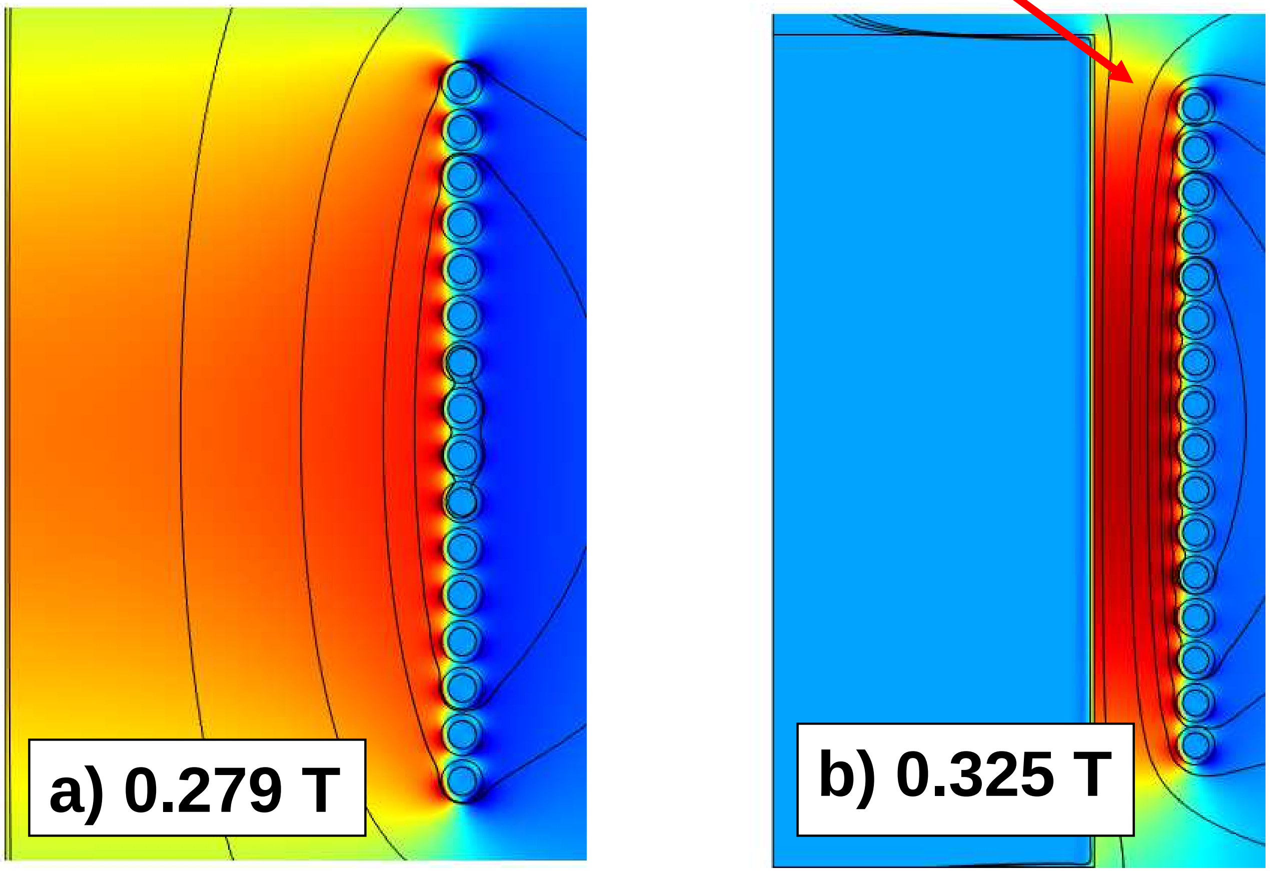
Modified

Short coil correction factor

'Air-Gap' Flux Density



A High Conductivity Work Piece at 10 kHz, Causing an ~16% increase in field strength in the 'air-gap'.



COMSOL® FEM

Modified Short Coil Correction Factors

Vaughan and Williamson (1945):

$$k_N^{\ *} = k_N\left(1-\left(\frac{D_w}{D_c}\right)^2\right)+\left(\frac{D_w}{D_c}\right)^2$$

Nagaoka Short Coil Correction Factor

Kennedy *et al.* (2011):

$$k_N^{\ *} = k_N\left(1-\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2\right)+\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2$$

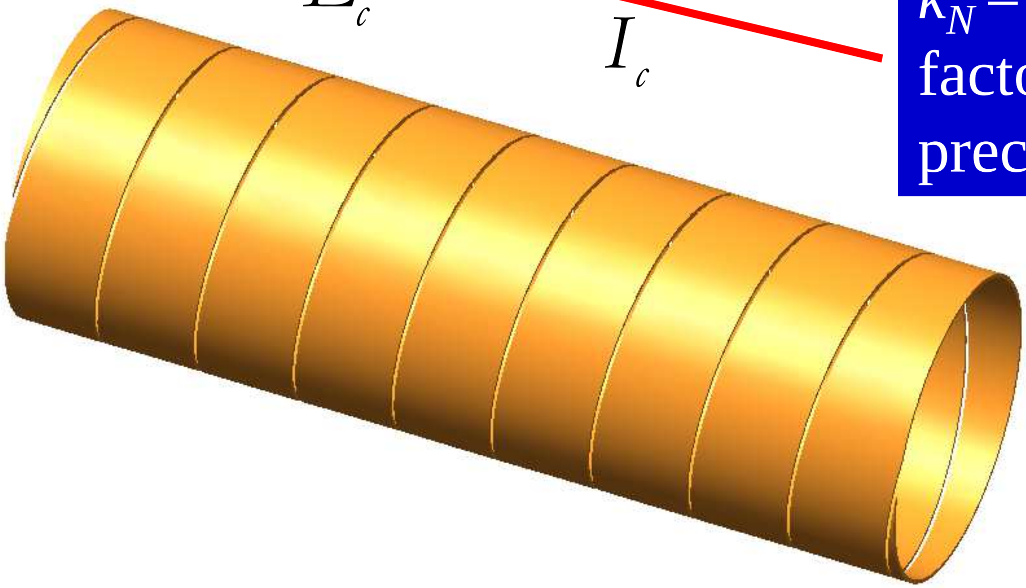
A validated FEM model was required to explore the analytical model accuracy vs. frequency.

FEM Model - How to find the required magnetic domain size, i.e. how big does the Universe need to be?

$$L_c = \frac{8\mu_0 \mu_r N_c^2 r_{cs}^3}{3 l_c^2} \left[\frac{2k^2 - 1}{k^3} E(k) + \frac{1 - k^2}{k^3} K(k) - 1 \right] \qquad k = \sqrt{\frac{4 r_{cs}^2}{4 r_{cs}^2 + l_c^2}}$$

$$L_c = \frac{k_N A_c N_c B_\infty}{I_c}$$

k_N = Nagaoka short coil correction factor. Can be solved to double precision accuracy numerically.



$$B_\infty = \frac{\mu_o \mu_r N_c I_c}{l_c}$$

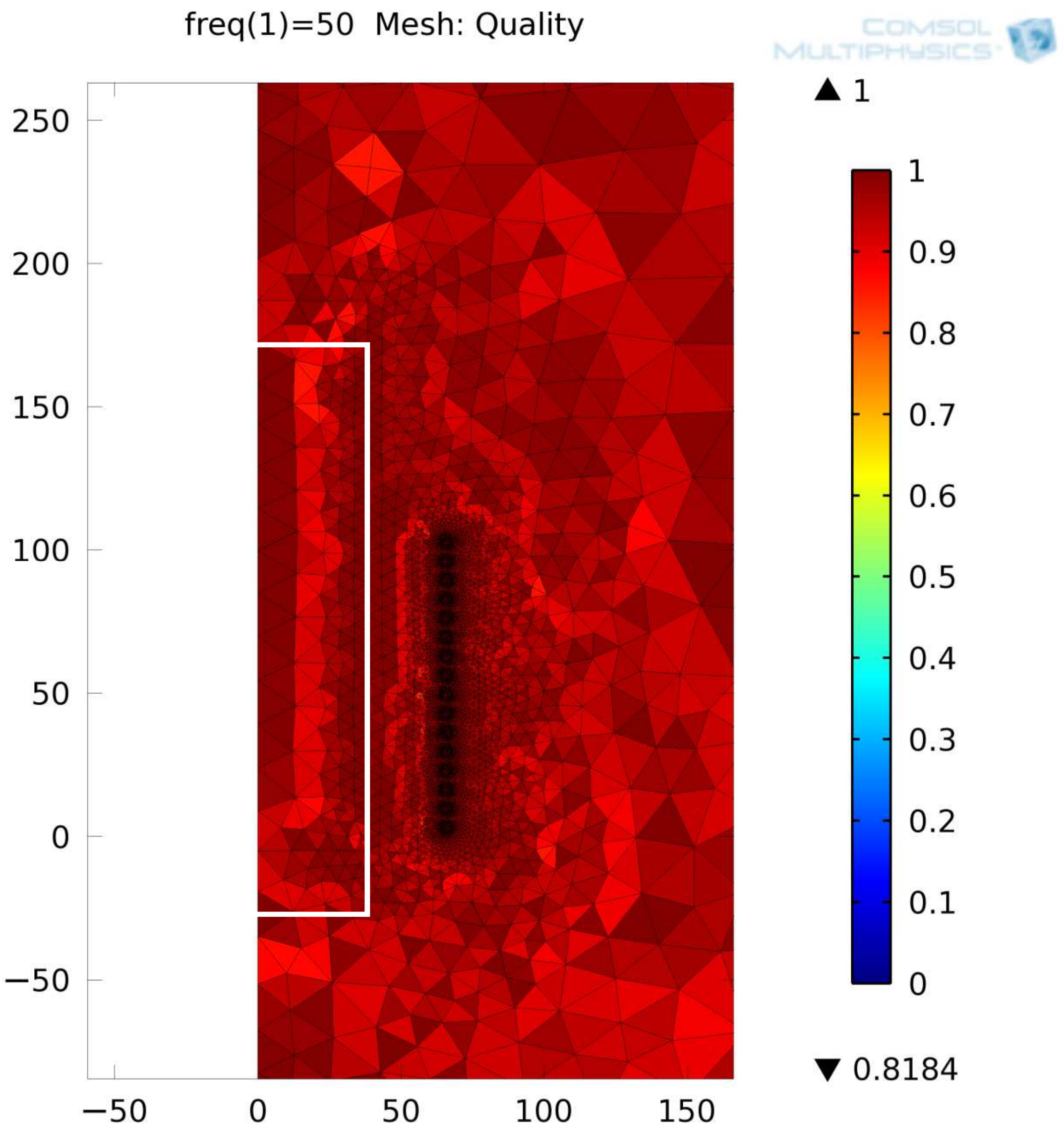
Comparison of FEM and Analytical Inductance of a Current Sheet

Ratio of Magnetic Domain Dimensions to Coil Dimensions	COMSOL Calculated Inductance (µH)	COMSOL - Analytical Solution Difference (%)
2.00	22.7563	-13.82
4.00	25.9502	-1.72
6.00	26.2783	-0.48
10.00	26.3870	-0.07
14.00	26.4057	0.00
20.00	26.4129	0.03

Error in inductance is the same as for the flux density and is then squared when calculating heating rate!

Theoretical answer = 26.4051 µH.
Ratio of 14 gives ideal results.

What is an Adequate Mesh?



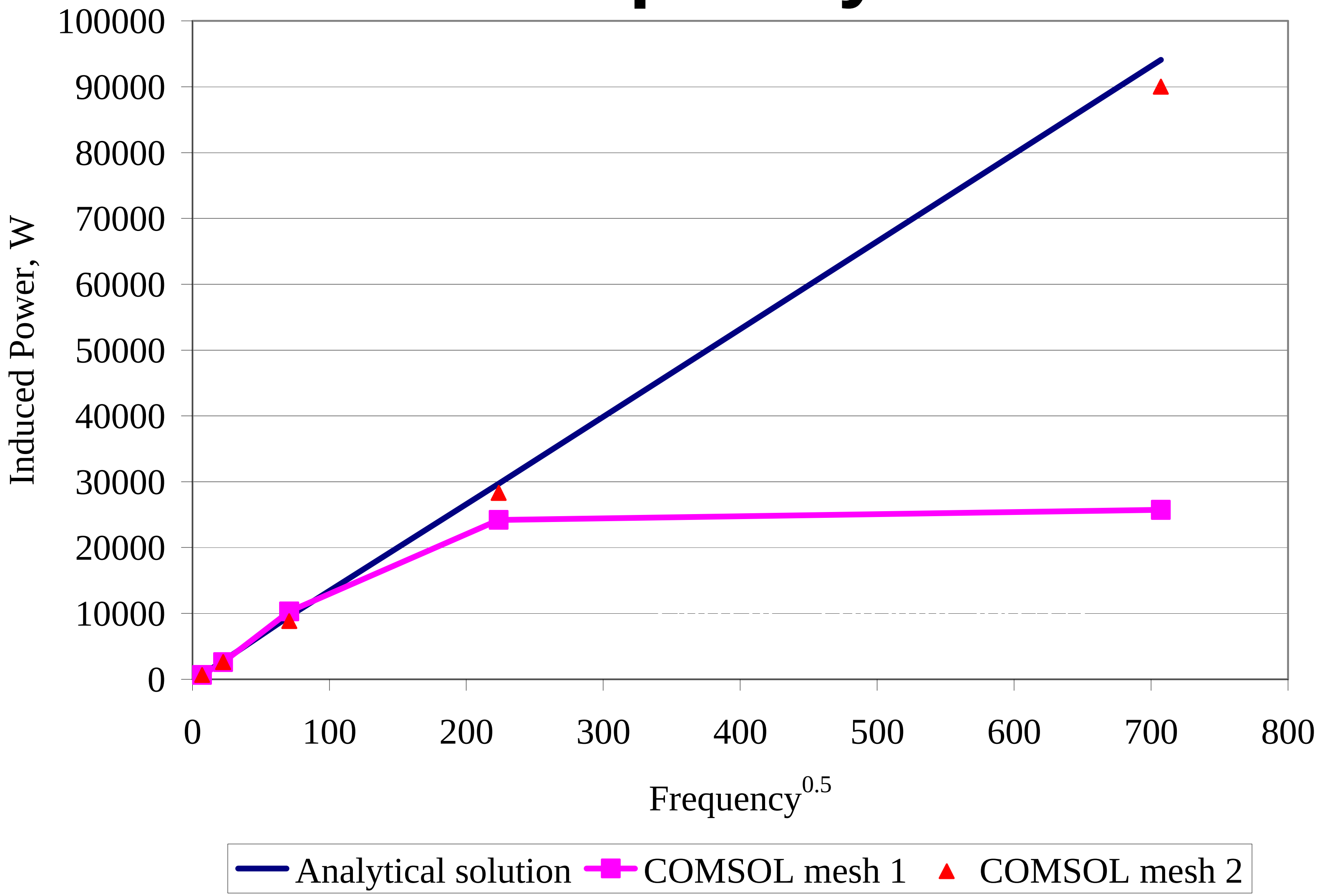
Induction Heating Using Mesh 1

Frequency (Hz)	Experimental Power (W)	Analytical Power (W)	Mesh 1 Power (W)	Mesh 1-Analytical Difference (%)	δ (mm)
50	696	691	650	-6.0	14.50
500	N/A	2768	2604	-5.9	4.59
5000	N/A	9549	10280	7.7	1.45
50000	N/A	29697	24211	-18.5	0.46
500000	N/A	94123	25728	-72.7	0.14
	Mesh 1spacing at work piece interface =				5.10

At 'High Frequency' the power induced should change by $\sqrt{f}$.

The first electromagnetic penetration depth will contain **63%** of the total current and **86%** of the power, with an exponential gradient **squared**.

Variation of Heating Rate with Frequency$^{0.5}$



Induction Heating Using Boundary Meshes

Frequency (Hz)	Experimental Power (W)	Analytical Power (W)	Mesh 2 Power (W)	Mesh 2-Analytical Difference (%)	δ (mm)
50	696	691	650	-6.0	14.5
500	N/A	2768	2597	-6.2	4.59
5000	N/A	9549	8834	-7.5	1.45
50000	N/A	29697	28305	-4.7	0.46
500000	N/A	94123	90029	-4.3	0.14
	Mesh 2 spacing at work piece interface =				0.02

Boundary meshes allow accurate calculation to extremely high frequency. Mesh spacing should be < δ.

High Precision Instrumentation

Electrical conductivity accuracy of +/- 0.5%
Standards +/- 0.01% IACS

Magnetic field measurements
Axial/Transverse
From 0.1μT-30T
+/- 1% AC
Standards from 500-2000 Gauss

Electrical analysis:

1. V, I, P (+/-100 W), p.f.
2. Inductance
3. Harmonics
4. Current +/- 1% (usable up to 100 kHz)

Work Piece Electrical Resistivity vs. Temperature

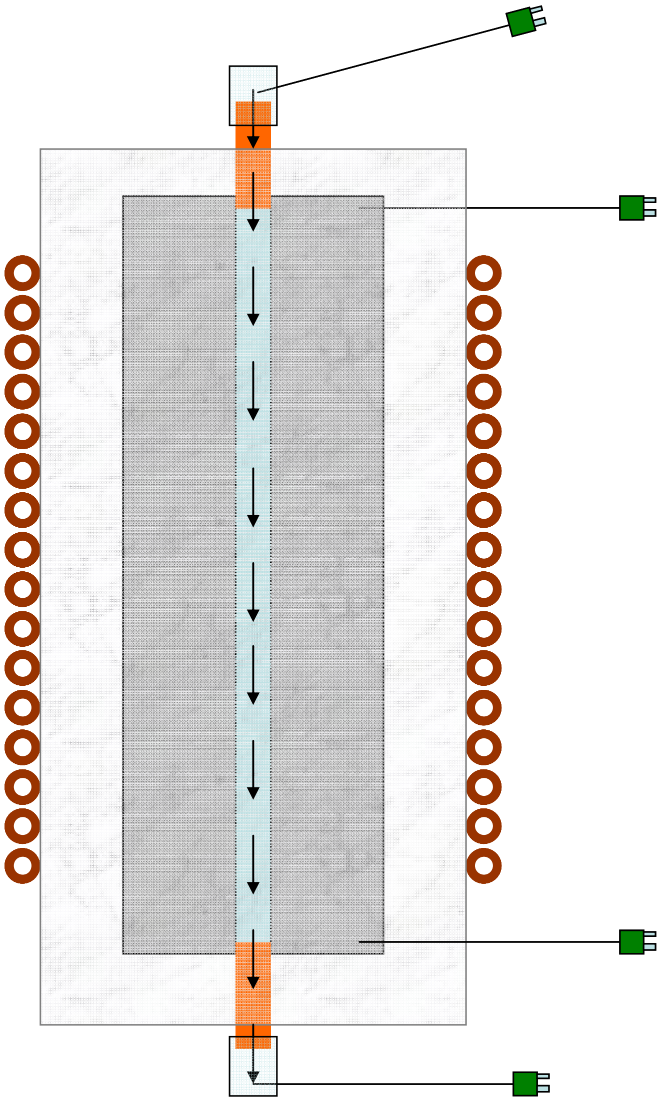
$$\rho = \rho_o \left(1 + \alpha_{293}\left[T - 293K\right]\right)$$

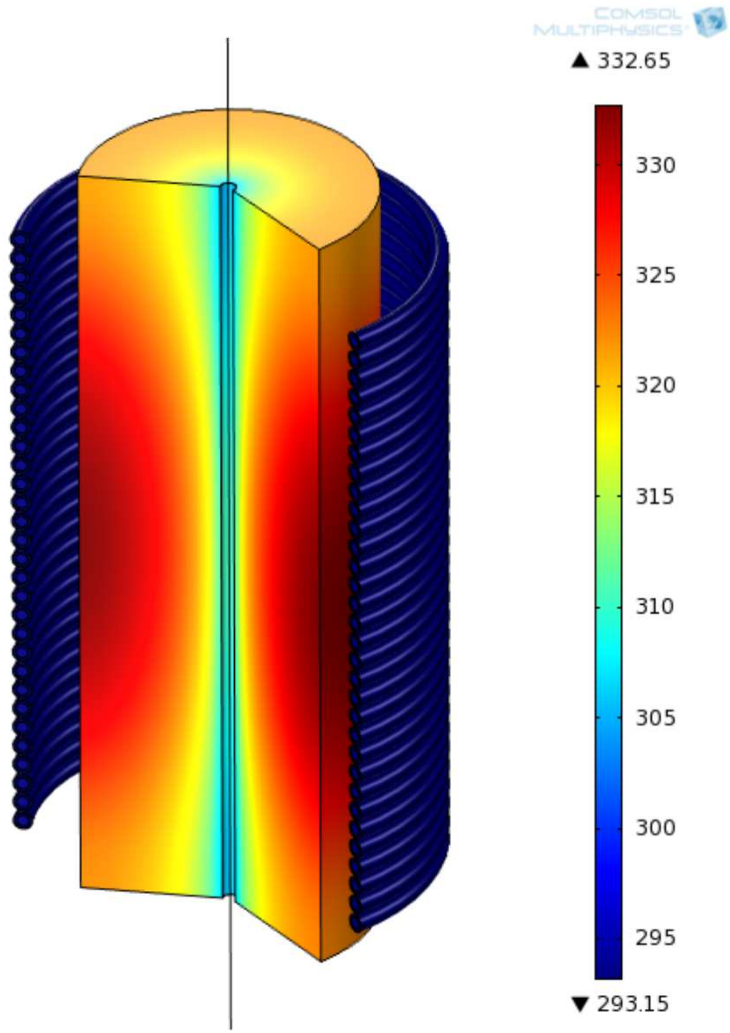
$$\rho_{0 \text{ for Alloy XXXX}} = \frac{\rho_{0 \text{ (for 100\% IACS Copper)}} \; 100\%}{\% \text{ IACS for Alloy XXXX}}$$

$$\alpha_{293 \textit{ for Alloy XXXX}} = \frac{0.0043 \text{ (Alloy XXXX \%IACS)}}{65.0}$$

Resistivity can then be found for any alloy from a single reading at room temperature.

Experimental Apparatus

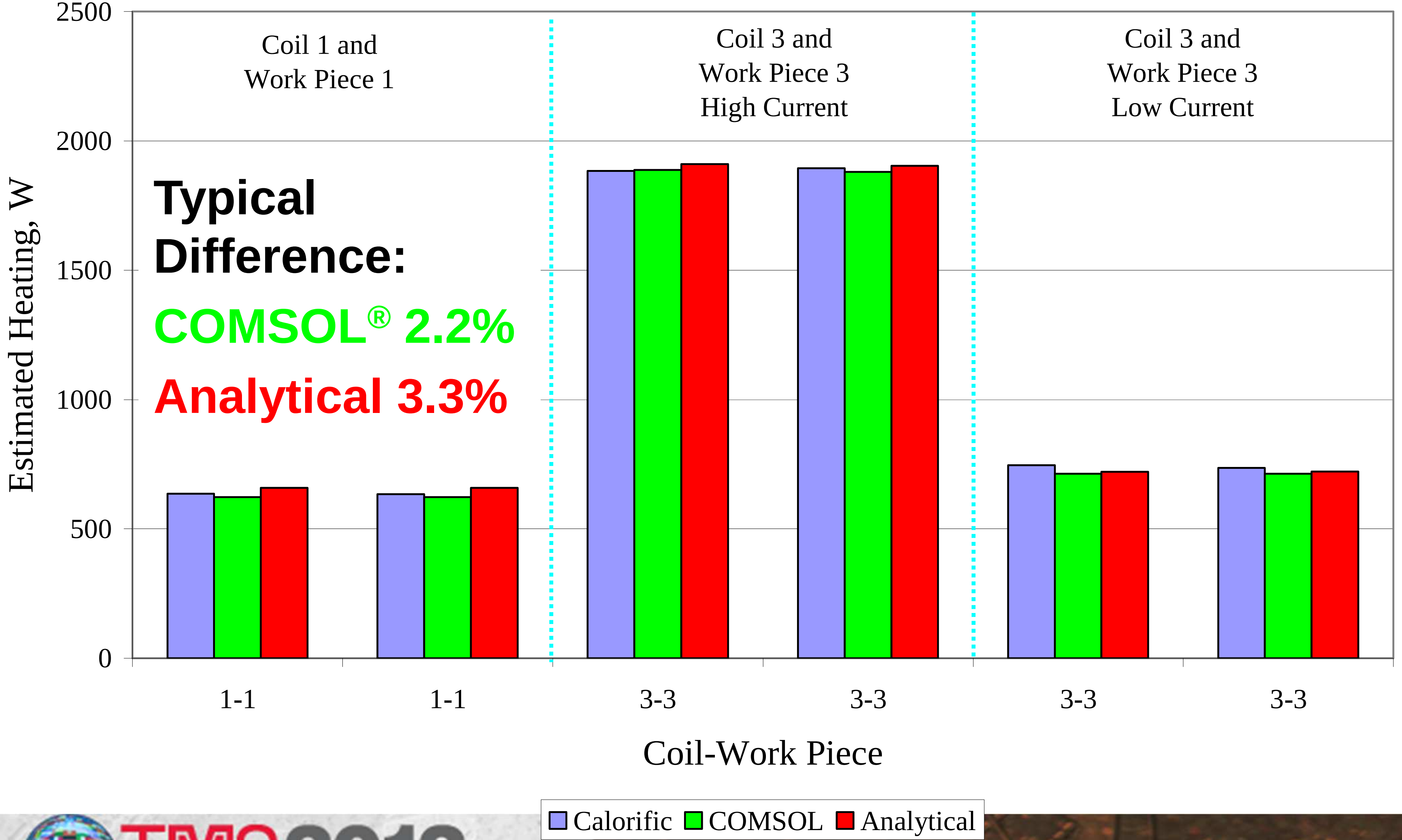




Work Pieces and Coils

Work Pieces	1	3
Alloy	A356	6060
Diameter, mm	75.0	95.0
Length, mm	130.0	260.0
Measured IACS Electrical Conductivity, %	48.4	53.4
Penetration Depth δ_w (mm) at 50 Hz and 293 K	13.43	12.79
ξ_w	3.948	5.252
$\varphi(\xi_w)$	0.823	0.859
Coil 1	1-1	
Coil 3		3-3
Coils	**Short Coil 1**	**Long Coil 3**
Average Diameter, mm	132	132
Height, mm	106	218
Diameter to Height ratio	1.24	0.60
Number of Turns	16	32
Short Coil Correction Factor	0.641	0.786
Electrically Determined IACS Conductivity, %	80	80
Penetration Depth δ_c (mm) at 50 Hz and 293 K	10.45	10.45
Modified Nagaoka Coefficient k_N* for Work Piece 1	0.720	
Modified Nagaoka Coefficient k_N* for Work Piece 3		0.870

Comparison of Measured and Predicted Heating Rates at 50 Hz



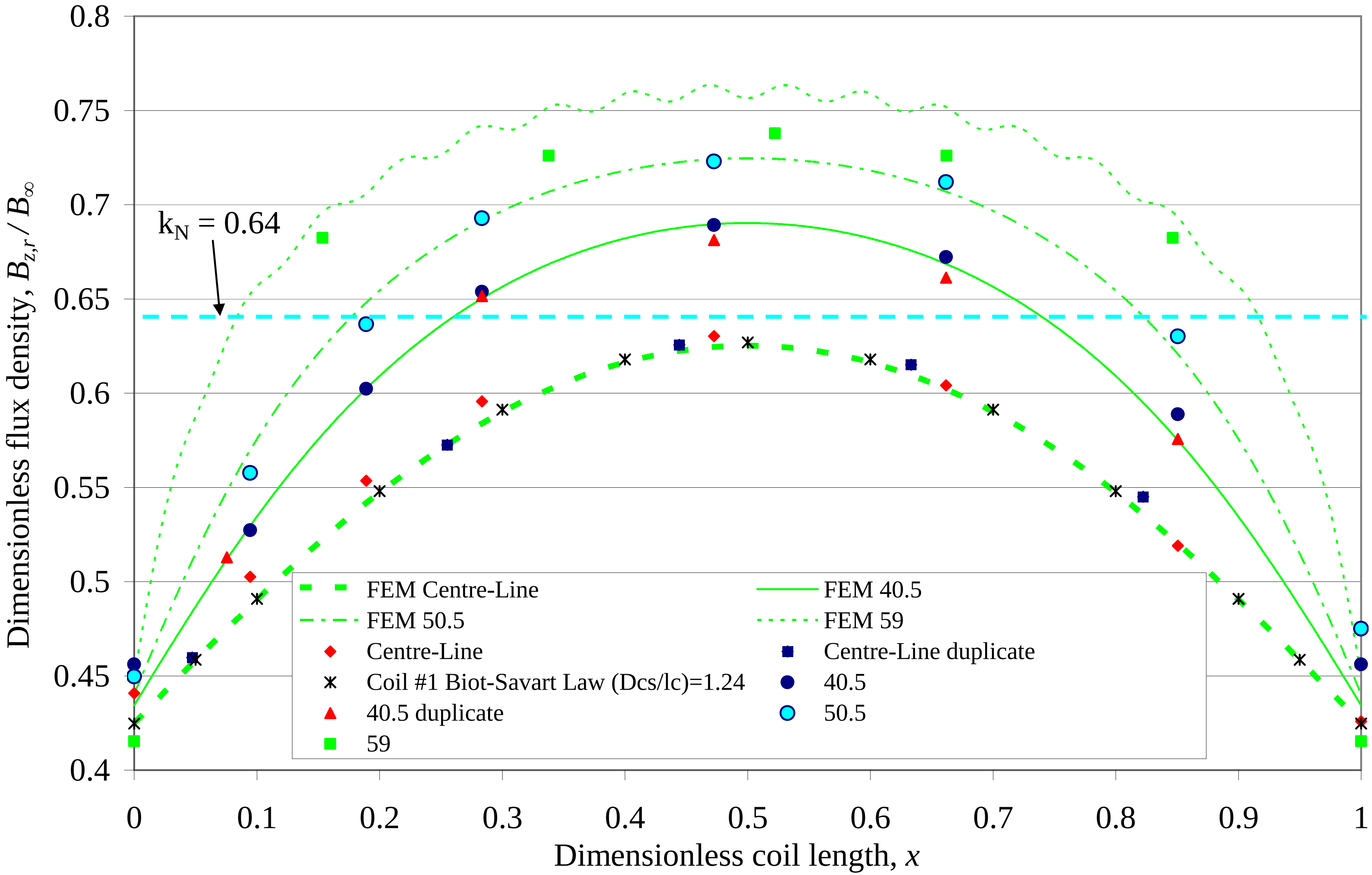
Induced Power vs. Frequency

Comparison between the Estimates of Power as a Function of Frequency for Coil #3 and Work piece #3

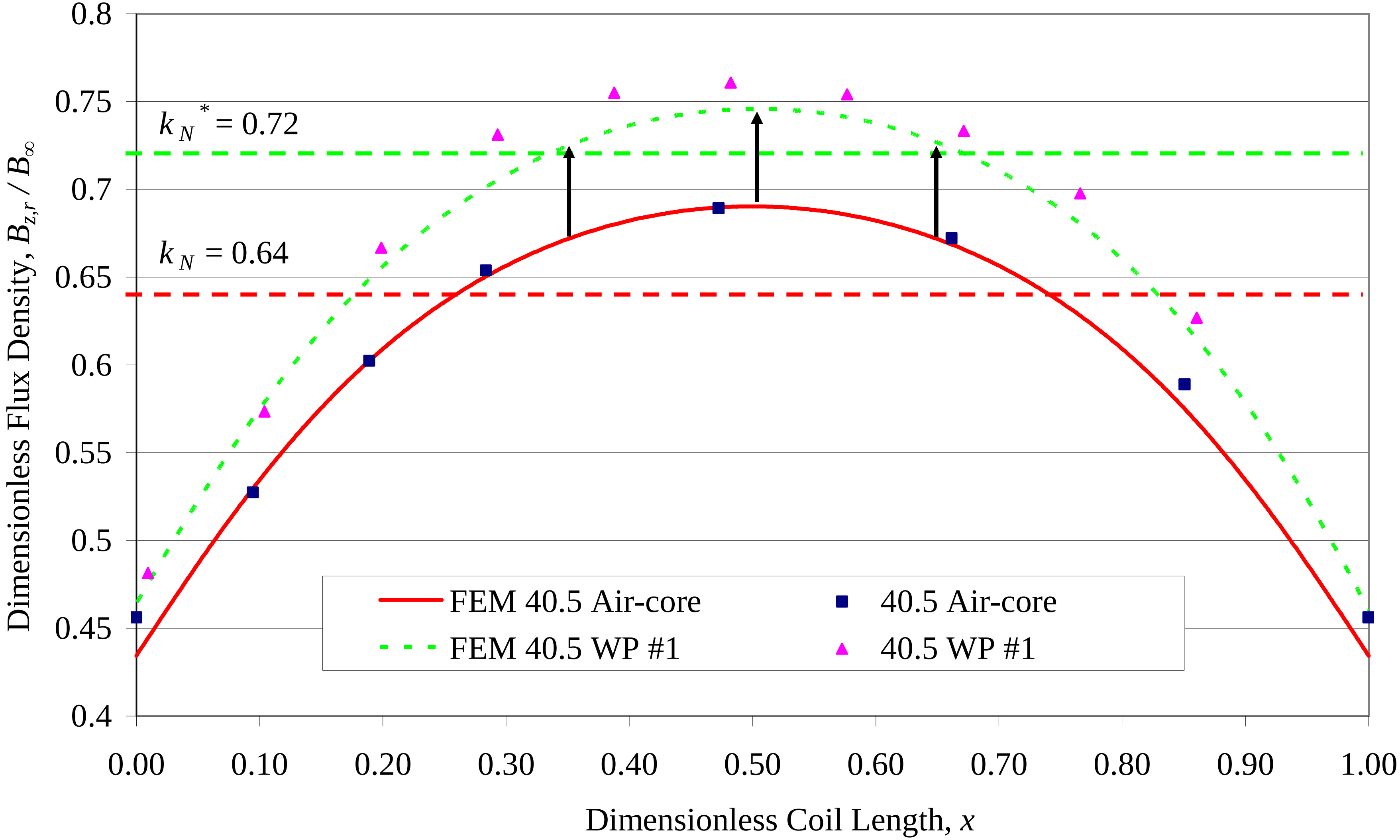
Frequency (Hz)	Thermal Experimental Power (W)	Electrical Experimental Power (W)	Analytical Power (W)	COMSOL Power (W)	Analytical-COMSOL Difference (%)
50	736	727	722	713	1.3
500	N/A	N/A	2660	2616	1.7
5000	N/A	N/A	8739	8704	0.4
50000	N/A	N/A	27762	27844	0.3
500000	N/A	N/A	87920	88348	0.5
				Average:	0.8

Consistent difference from low-high frequency

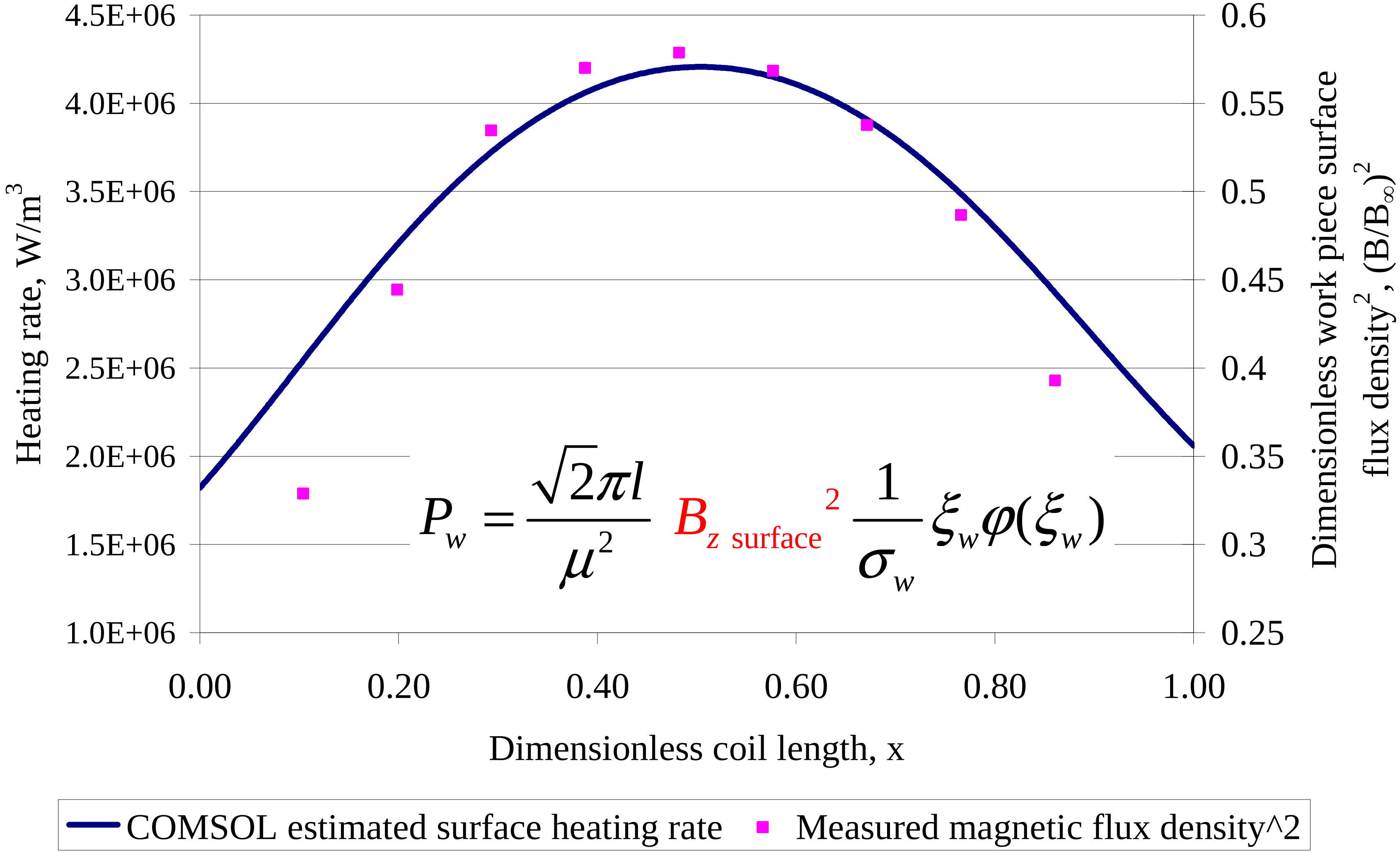
Dimensionless Flux Density vs. Position



Dimensionless Flux Density vs. With and Without a Work Piece

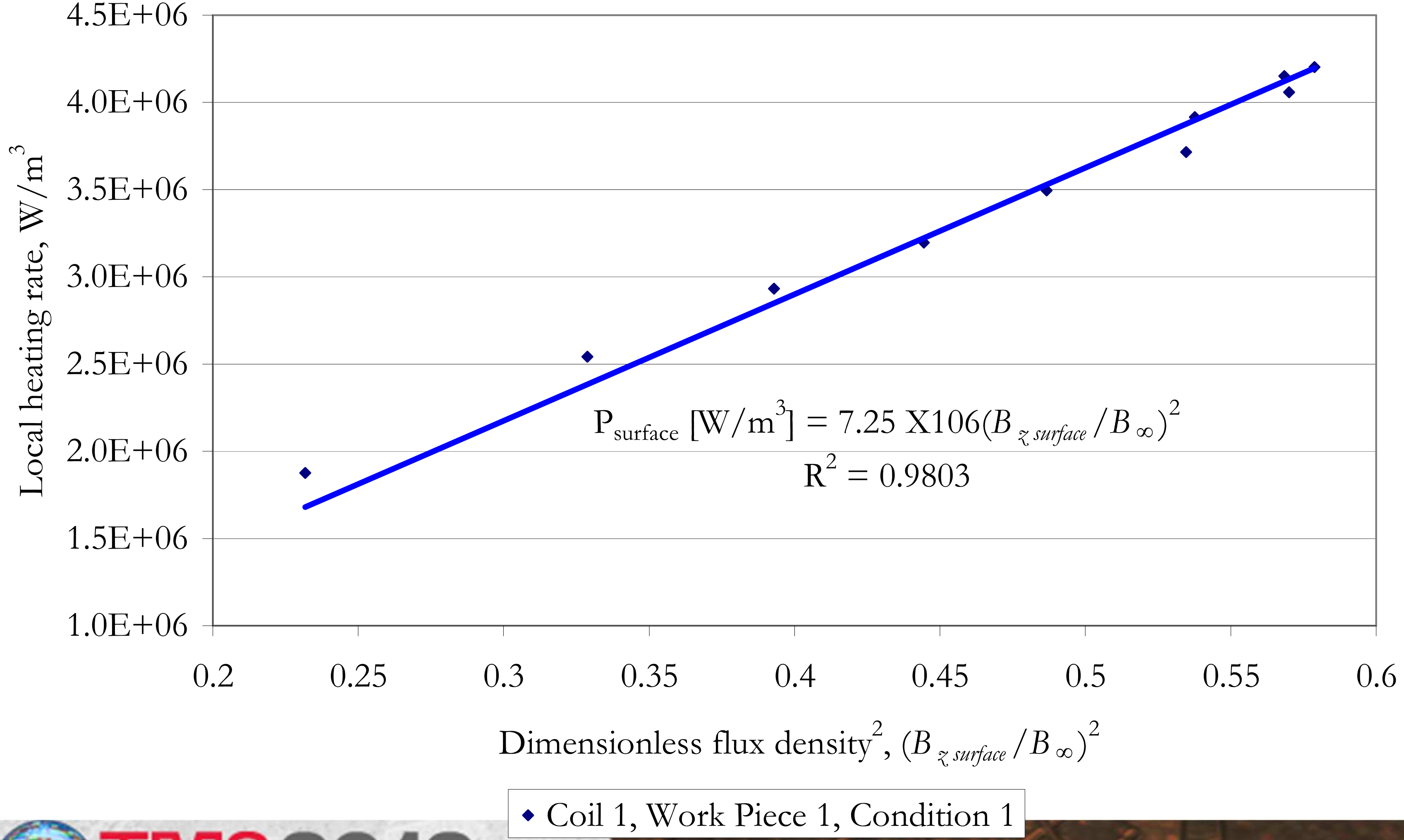


Variation of Heating Rate with Coil Position and Flux Density



$$P_w = \frac{\sqrt{2}\pi l}{\mu^2} B_{z\ surface}^{2} \frac{1}{\sigma_w} \xi_w \varphi(\xi_w)$$

Variation of Heating Rate with the Square of the Flux Density



Conclusions

- Accurate measurements and comparison with analytical solutions has allowed for precise validation of the COMSOL 4.2® 2D axial symmetric FEM model.

- Excellent agreement was achieved between data and model estimates of both the magnetic flux density and heating rate (induced power) at 50 Hz.

- Analytical and FEM model heating estimates were in consistent agreement at frequencies from 50 Hz to 500 kHz.

Conclusions

- The modified ‘short coil’ correction factor (Kennedy *et al*.) was a key factor in obtaining accurate analytical model predictions.

- A new method of correlating the resistivity temperature coefficient has been presented for use with aluminum and its alloys.

Acknowledgements

The present study was carried out as part of the RIRA (Remelting and Inclusion Refining of Aluminium) project funded by the Norwegian Research Council (NRC) - BIP Project No. 179947/I40.

The industrial partners involved in the project are: Hydro Aluminium AS, SAPA Heat Transfer AB, Alcoa Norway ANS, Norwegian University of Science and Technology (NTNU) and SINTEF Materials and Chemistry.

The funding granted by the industrial partners and the NRC is gratefully acknowledged.

Egil Torsetnes at NTNU for helping with the design and construction of the experimental apparatus. Kurt Sandaunet at SINTEF for his support and help, as well as for the use of the SINTEF laboratory.

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