ANALYTICAL AND FEM MODELING OF ALUMINUM BILLET INDUCTION HEATING WITH EXPERIMENTAL VERIFICATION

NTNU
Innovation and Creativity

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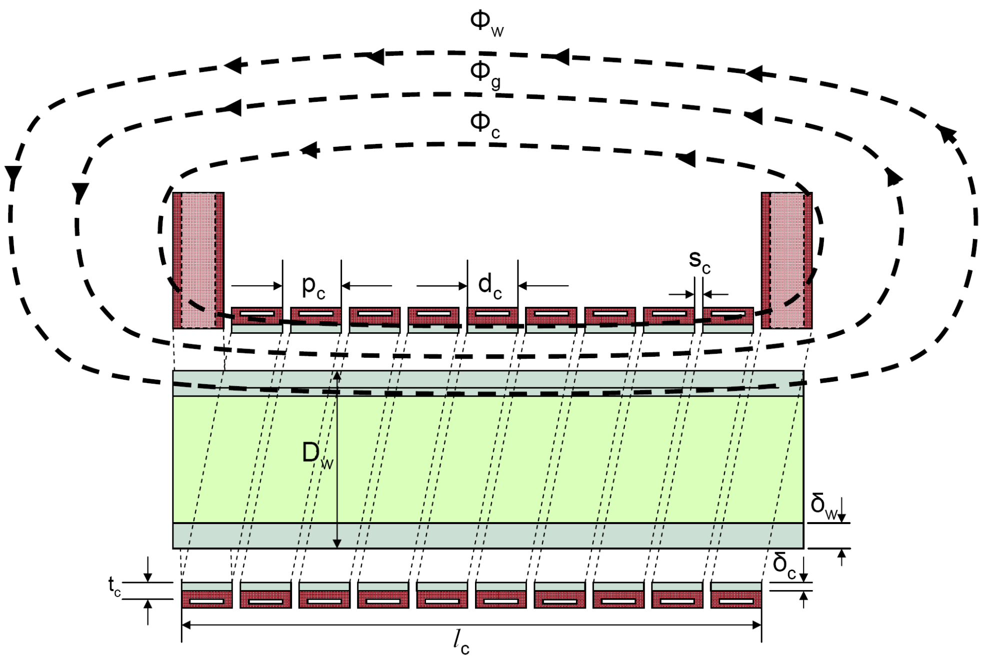
Heating will vary by the square of the flux density in the air-gap.

$$P_w = \frac{\sqrt{2}\pi l}{\mu^2}\left(\boxed{\frac{k_N^{*}\mu I_c N_c}{l_c}}\right)^2 \frac{1}{\sigma_w}\xi_w\varphi(\xi_w)$$

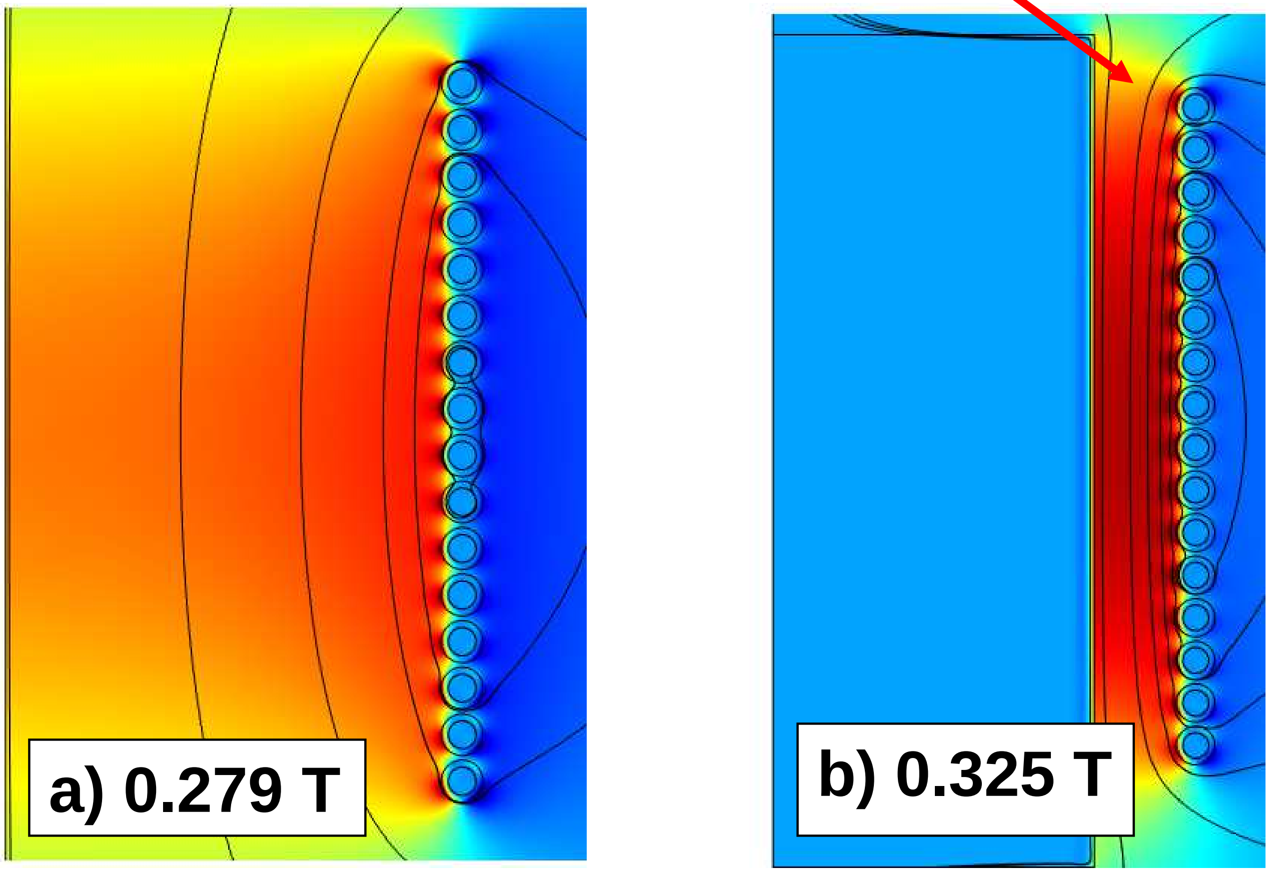
Modified

Short coil correction factor

'Air-Gap' Flux Density



A High Conductivity Work Piece at 10 kHz, Causing an ~16% increase in field strength in the 'air-gap'.



COMSOL® FEM

Modified Short Coil Correction Factors

Vaughan and Williamson (1945):

Nagaoka Short Coil Correction Factor

$$k_N^{\ *} = k_N\left(1-\left(\frac{D_w}{D_c}\right)^2\right)+\left(\frac{D_w}{D_c}\right)^2$$

Kennedy *et al.* (2011):

$$k_N^{\ *} = k_N\left(1-\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2\right)+\left(\frac{D_w-\delta_w}{D_c+\delta_c}\right)^2$$

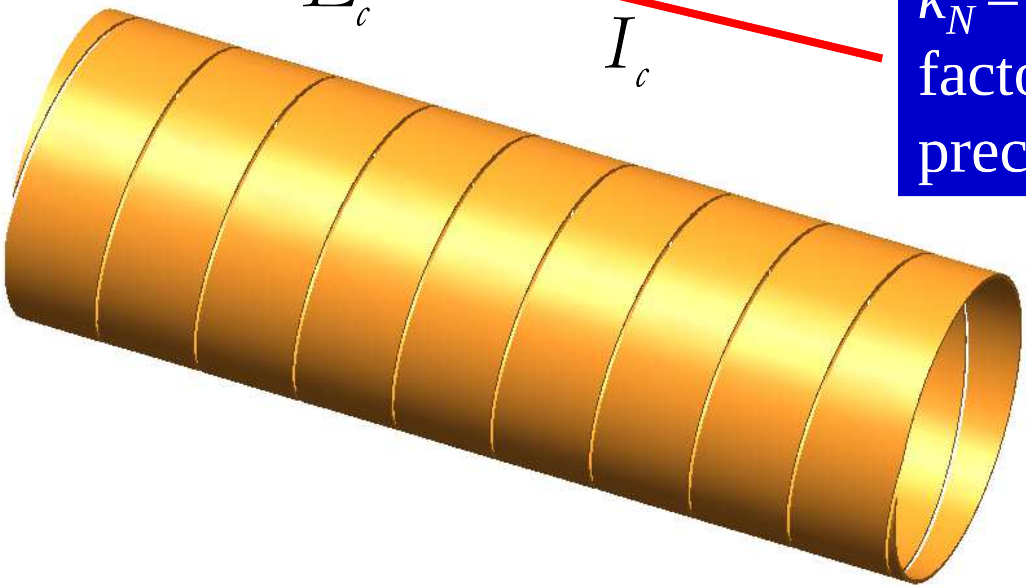
A validated FEM model was required to explore the analytical model accuracy vs. frequency.

FEM Model - How to find the required magnetic domain size, i.e. how big does the Universe need to be?

$$L_c = \frac{8\mu_0\mu_r N_c^{\,2} r_{cs}^{\,3}}{3l_c^{\,2}}\left[\frac{2k^2-1}{k^3}E(k)+\frac{1-k^2}{k^3}K(k)-1\right] \qquad k=\sqrt{\frac{4r_{cs}^{\,2}}{4r_{cs}^{\,2}+l_c^{\,2}}}$$

$$L_c = \frac{k_N A_c N_c B_\infty}{I_c}$$

k_N = Nagaoka short coil correction factor. Can be solved to double precision accuracy numerically.



$$B_\infty = \frac{\mu_o \mu_r N_c I_c}{l_c}$$

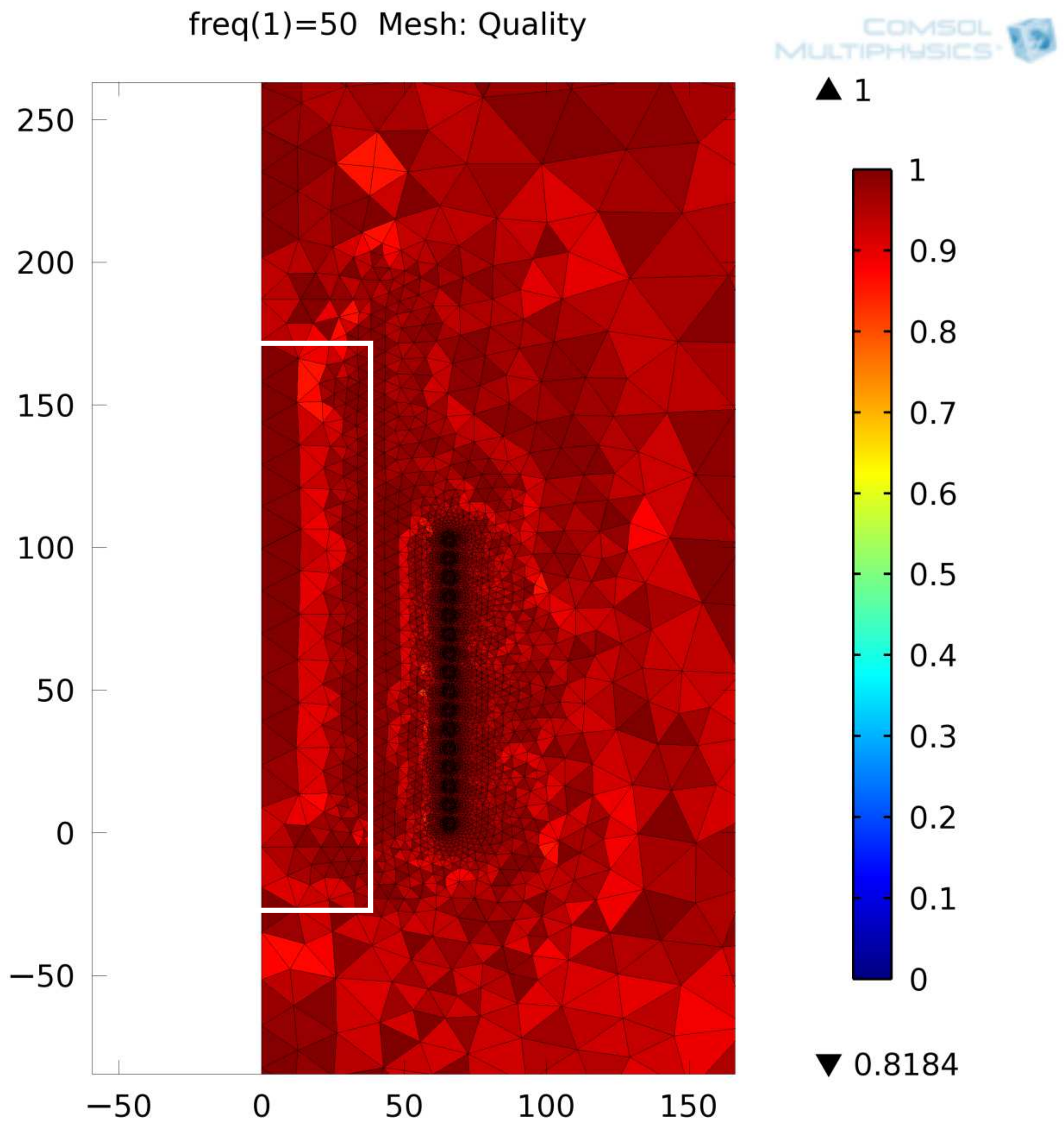
Comparison of FEM and Analytical Inductance of a Current Sheet

Ratio of Magnetic Domain Dimensions to Coil Dimensions	COMSOL Calculated Inductance (μH)	COMSOL - Analytical Solution Difference (%)
2.00	22.7563	-13.82
4.00	25.9502	-1.72
6.00	26.2783	-0.48
10.00	26.3870	-0.07
14.00	26.4057	0.00
20.00	26.4129	0.03

Error in inductance is the same as for the flux density and is then squared when calculating heating rate!

Theoretical answer = 26.4051 μH.
Ratio of 14 gives ideal results.

What is an Adequate Mesh?



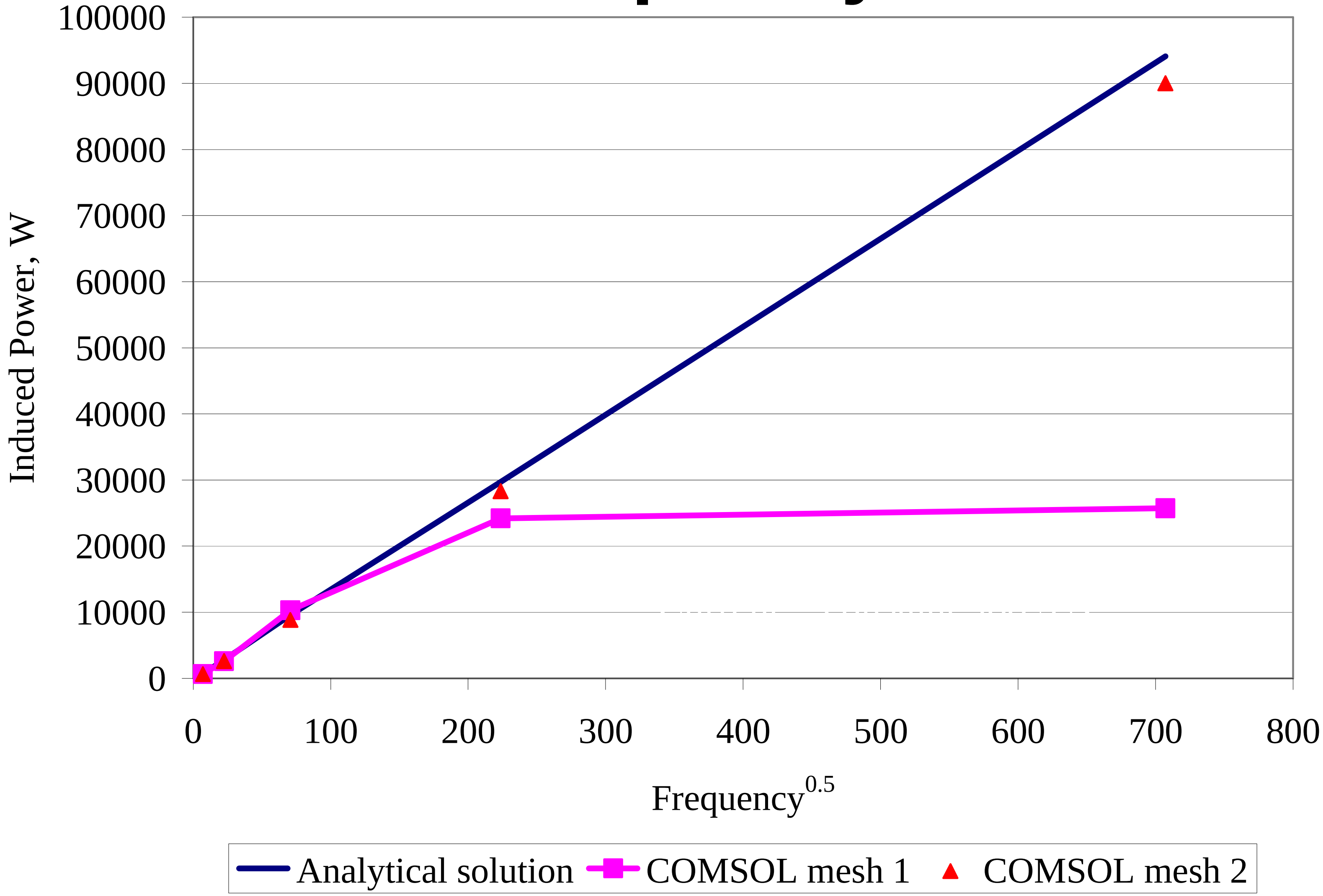
Induction Heating Using Mesh 1

Frequency (Hz)	Experimental Power (W)	Analytical Power (W)	Mesh 1 Power (W)	Mesh 1- Analytical Difference (%)	δ (mm)
50	696	691	650	-6.0	14.50
500	N/A	2768	2604	-5.9	4.59
5000	N/A	9549	10280	7.7	1.45
50000	N/A	29697	24211	-18.5	0.46
500000	N/A	94123	25728	-72.7	0.14
	Mesh 1spacing at work piece interface =				5.10

At 'High Frequency' the power induced should change by $\sqrt{f}$.

The first electromagnetic penetration depth will contain **63%** of the total current and **86%** of the power, with an exponential gradient **squared**.

Variation of Heating Rate with Frequency$^{0.5}$



Induction Heating Using Boundary Meshes

Frequency (Hz)	Experimental Power (W)	Analytical Power (W)	Mesh 2 Power (W)	Mesh 2-Analytical Difference (%)	δ (mm)
50	696	691	650	-6.0	14.5
500	N/A	2768	2597	-6.2	4.59
5000	N/A	9549	8834	-7.5	1.45
50000	N/A	29697	28305	-4.7	0.46
500000	N/A	94123	90029	-4.3	0.14
	Mesh 2 spacing at work piece interface =				0.02

Boundary meshes allow accurate calculation to extremely high frequency. Mesh spacing should be < δ.

High Precision Instrumentation

Electrical conductivity accuracy of +/- 0.5%
Standards +/- 0.01% IACS

Magnetic field measurements
Axial/Transverse
From 0.1μT-30T
+/- 1% AC
Standards from 500-2000 Gauss

Electrical analysis:

1. V, I, P (+/-100 W), p.f.
2. Inductance
3. Harmonics
4. Current +/- 1% (usable up to 100 kHz)

Work Piece Electrical Resistivity vs. Temperature

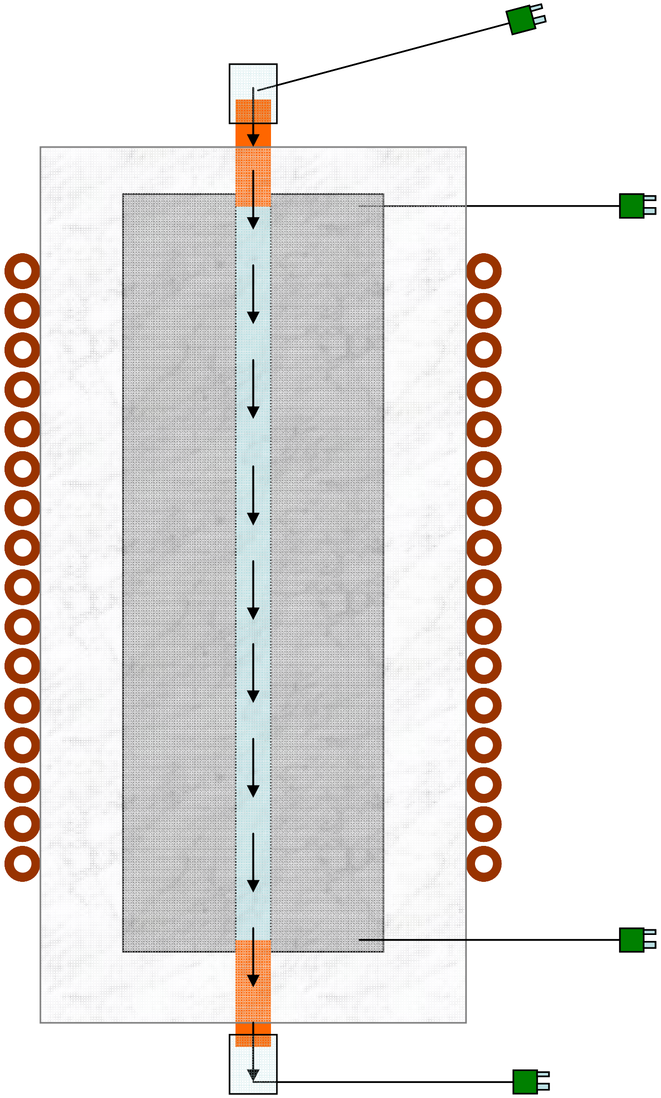
$$\rho = \rho_o \left(1 + \alpha_{293}\left[T - 293K\right]\right)$$

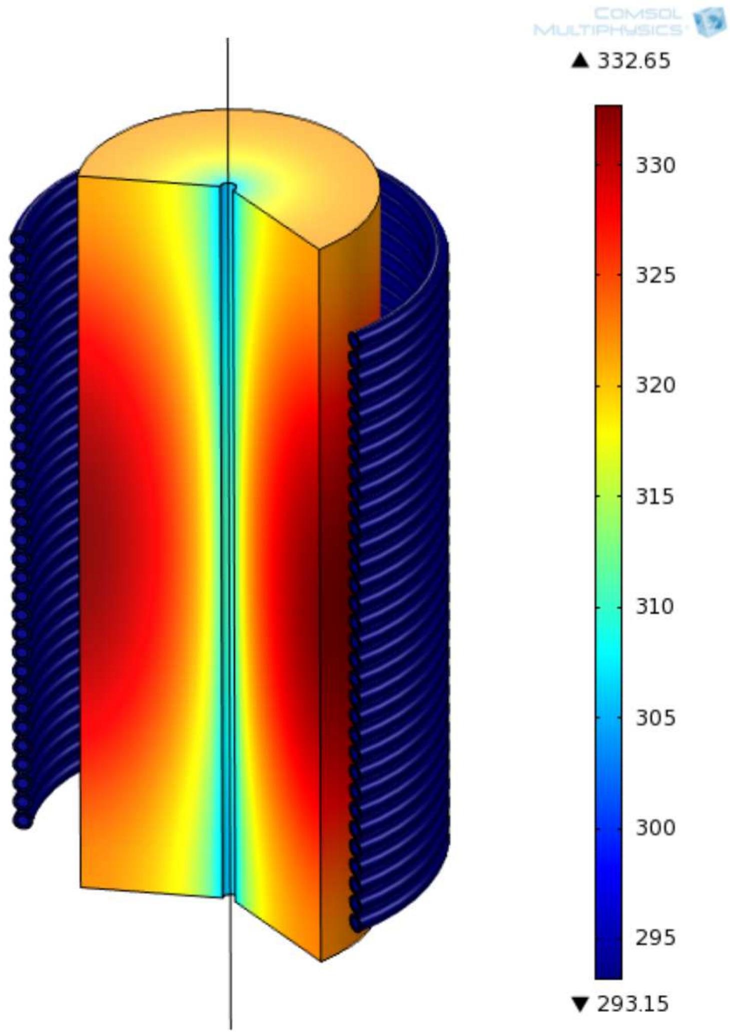
$$\rho_{0 \text{ for Alloy XXXX}} = \frac{\rho_{0 \text{ (for 100\% IACS Copper)}} \quad 100\%}{\text{\% IACS for Alloy XXXX}}$$

$$\alpha_{293 \ \textit{for Alloy XXXX}} = \frac{0.0043 \text{ (Alloy XXXX \%IACS)}}{65.0}$$

Resistivity can then be found for any alloy from a single reading at room temperature.

Experimental Apparatus

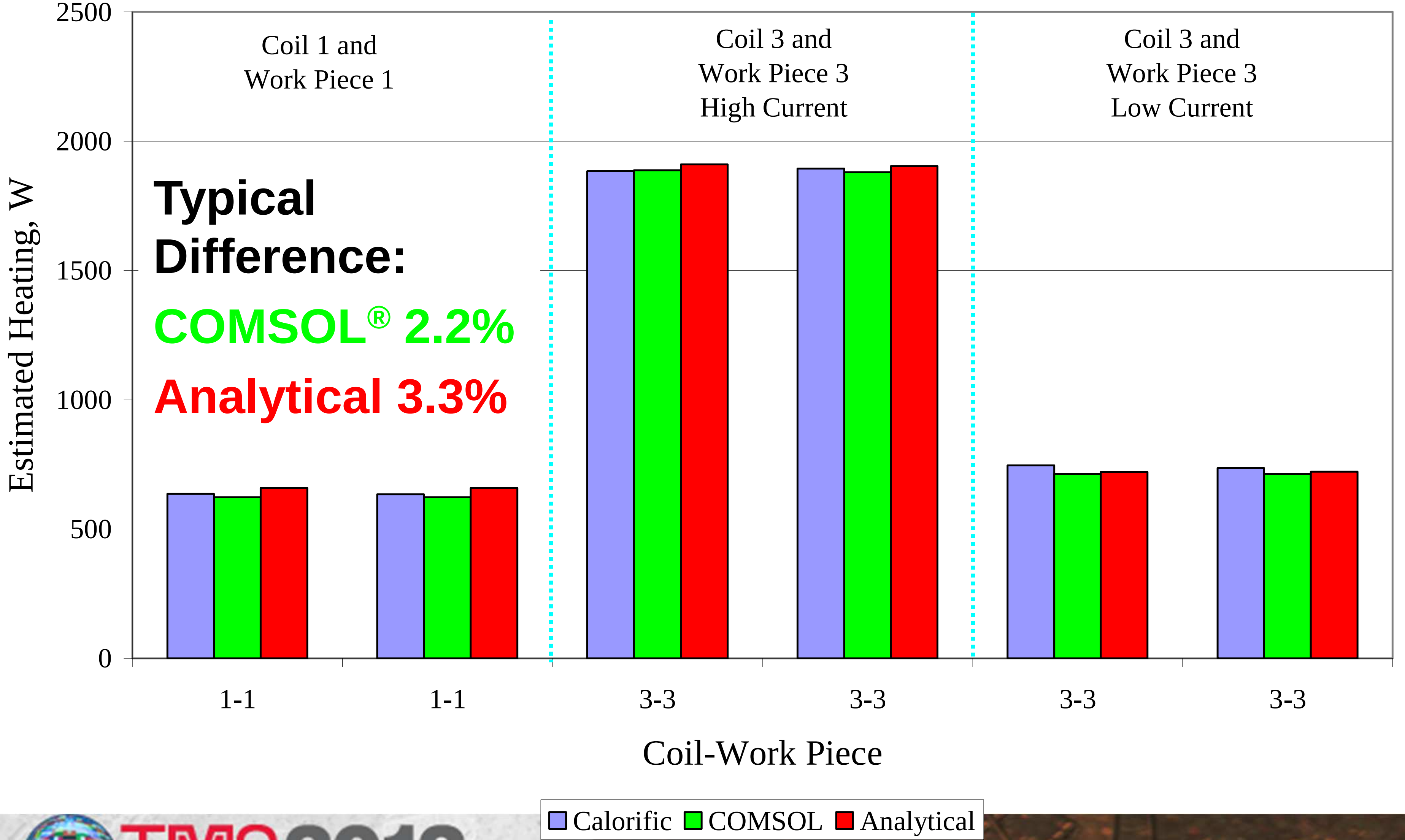




Work Pieces and Coils

Work Pieces	1	3
Alloy	A356	6060
Diameter, mm	75.0	95.0
Length, mm	130.0	260.0
Measured IACS Electrical Conductivity, %	48.4	53.4
Penetration Depth δ_w (mm) at 50 Hz and 293 K	13.43	12.79
ξ_w	3.948	5.252
$\varphi(\xi_w)$	0.823	0.859
Coil 1	1-1	
Coil 3		3-3
Coils	**Short Coil 1**	**Long Coil 3**
Average Diameter, mm	132	132
Height, mm	106	218
Diameter to Height ratio	1.24	0.60
Number of Turns	16	32
Short Coil Correction Factor	0.641	0.786
Electrically Determined IACS Conductivity, %	80	80
Penetration Depth δ_c (mm) at 50 Hz and 293 K	10.45	10.45
Modified Nagaoka Coefficient k_N* for Work Piece 1	0.720	
Modified Nagaoka Coefficient k_N* for Work Piece 3		0.870

Comparison of Measured and Predicted Heating Rates at 50 Hz



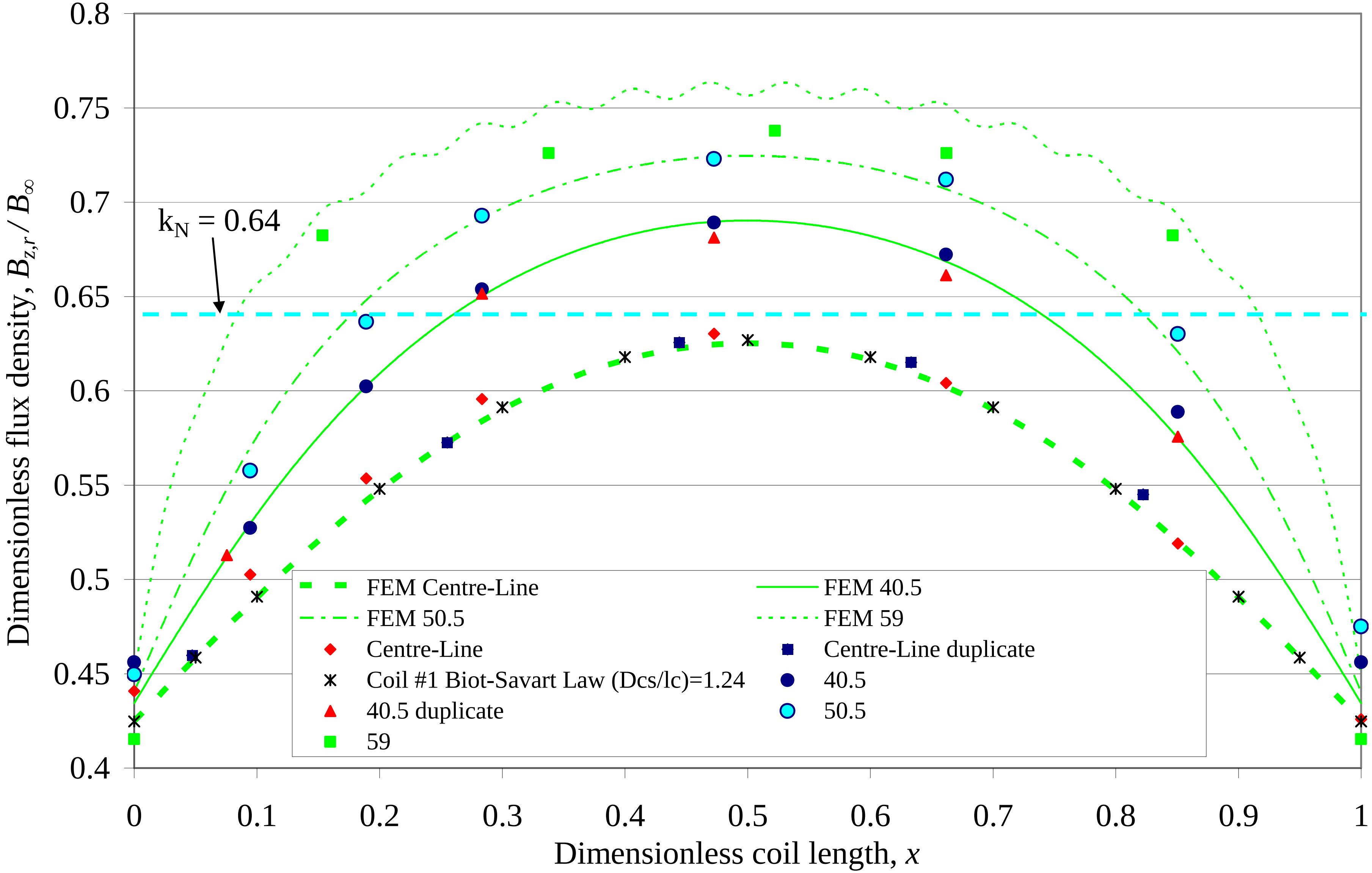
Induced Power vs. Frequency

Comparison between the Estimates of Power as a Function of Frequency for Coil #3 and Work piece #3

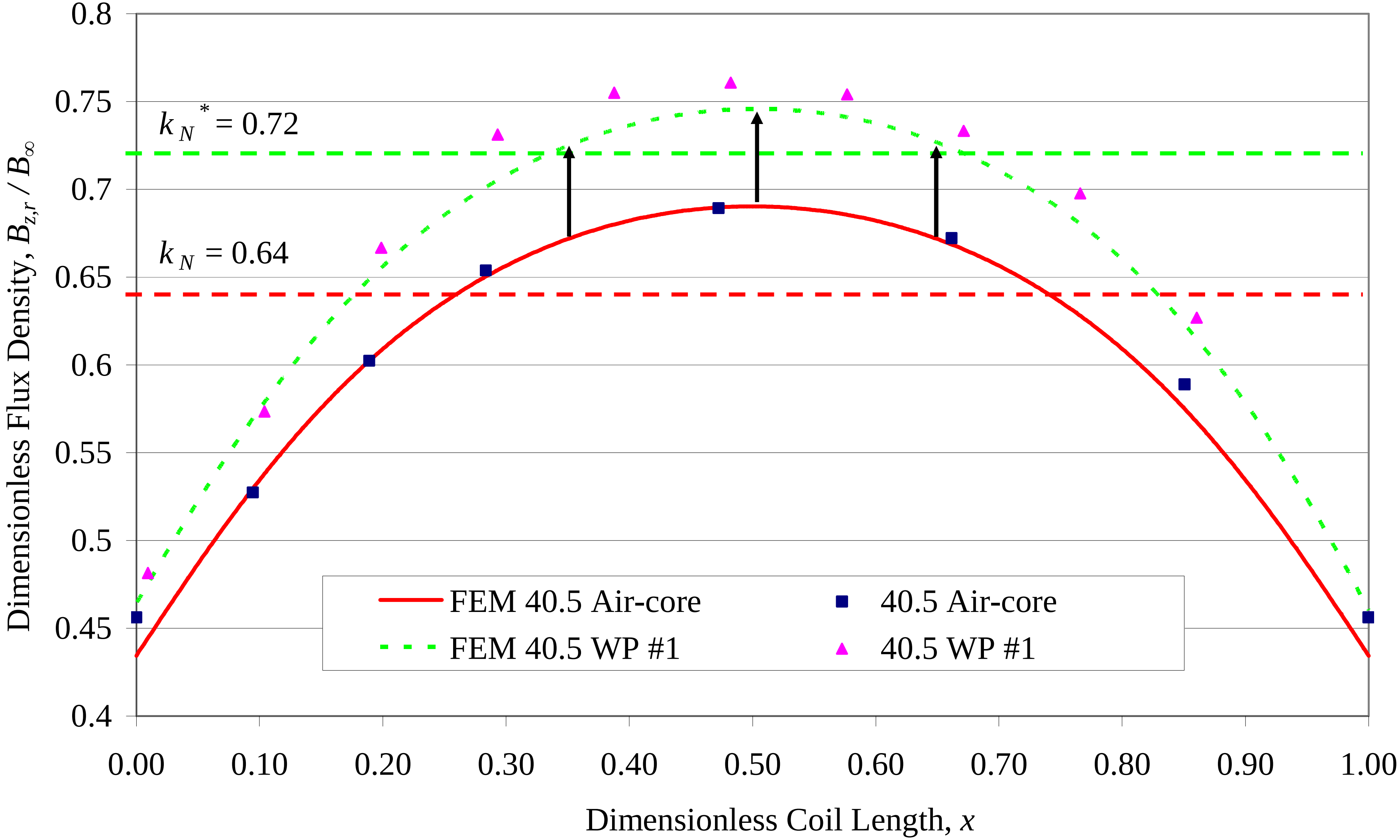
Frequency (Hz)	Thermal Experimental Power (W)	Electrical Experimental Power (W)	Analytical Power (W)	COMSOL Power (W)	Analytical-COMSOL Difference (%)
50	736	727	722	713	1.3
500	N/A	N/A	2660	2616	1.7
5000	N/A	N/A	8739	8704	0.4
50000	N/A	N/A	27762	27844	0.3
500000	N/A	N/A	87920	88348	0.5
				Average:	0.8

Consistent difference from low-high frequency

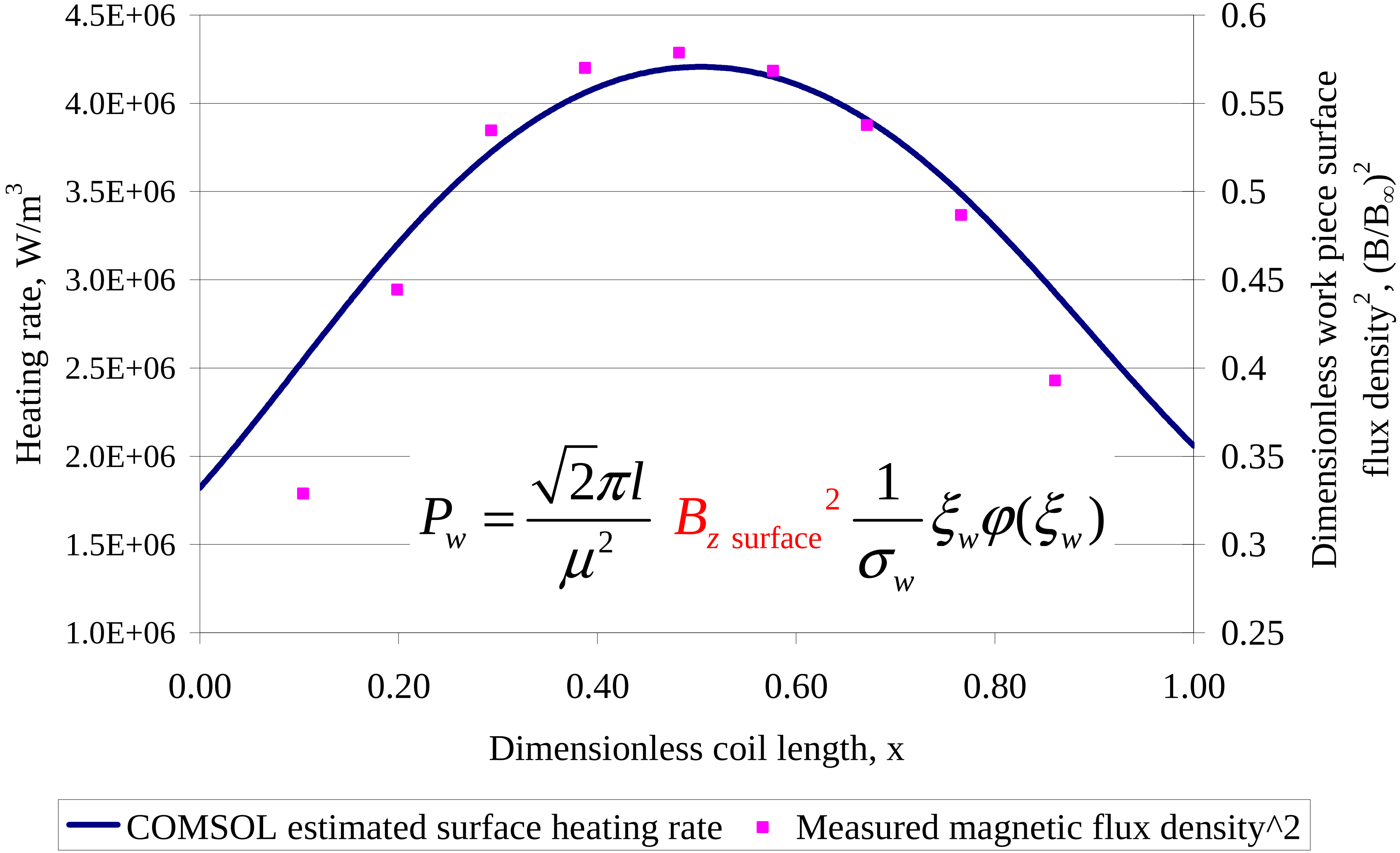
Dimensionless Flux Density vs. Position



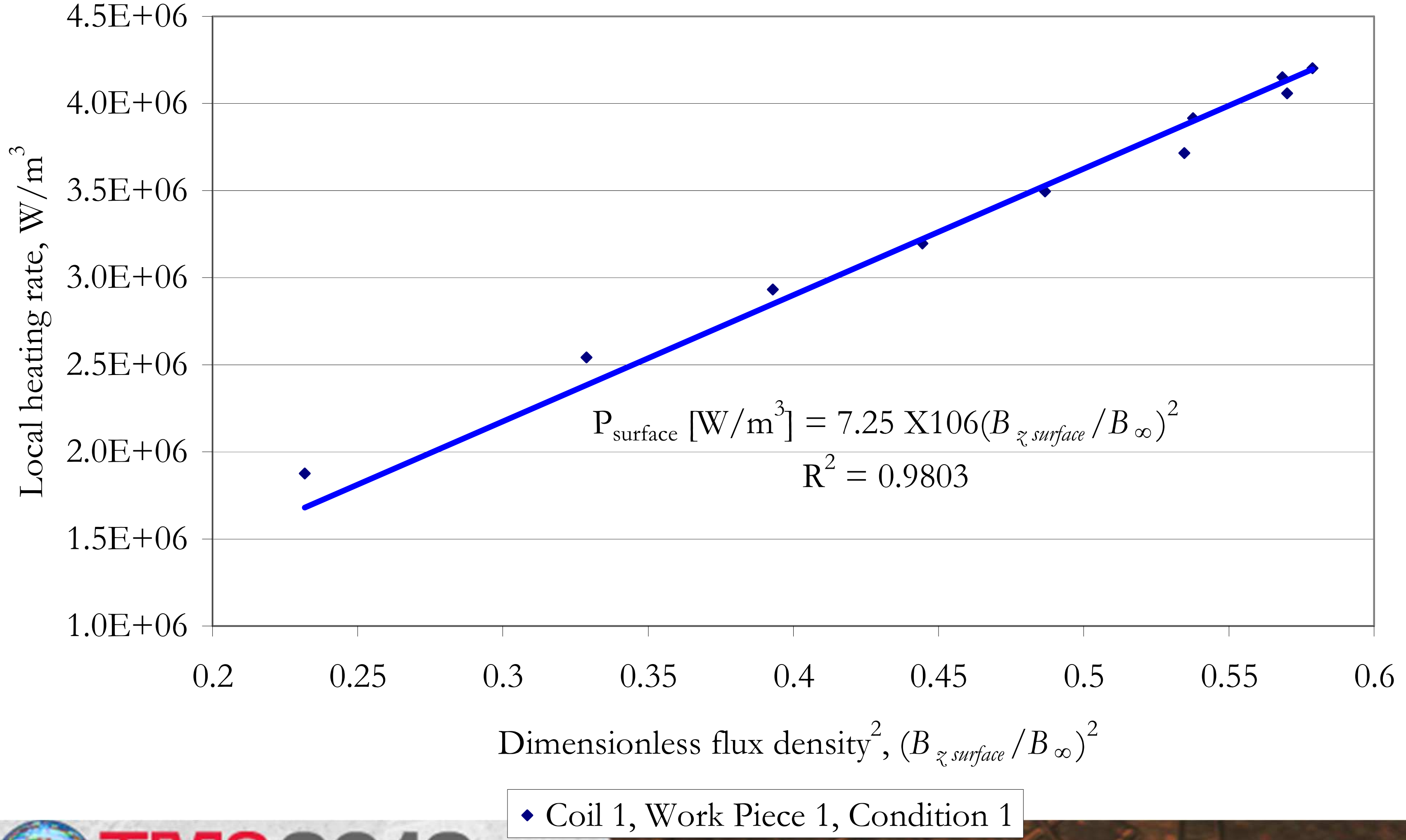
Dimensionless Flux Density vs. With and Without a Work Piece



Variation of Heating Rate with Coil Position and Flux Density



Variation of Heating Rate with the Square of the Flux Density



Conclusions

- Accurate measurements and comparison with analytical solutions has allowed for precise validation of the COMSOL 4.2® 2D axial symmetric FEM model.

- Excellent agreement was achieved between data and model estimates of both the magnetic flux density and heating rate (induced power) at 50 Hz.

- Analytical and FEM model heating estimates were in consistent agreement at frequencies from 50 Hz to 500 kHz.

Conclusions

- The modified ‘short coil’ correction factor (Kennedy *et al*.) was a key factor in obtaining accurate analytical model predictions.
- A new method of correlating the resistivity temperature coefficient has been presented for use with aluminum and its alloys.

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