Review of Classical Design Methods as Applied to Aluminum Billet Heating with Induction Coils

Effect of coil and work piece geometry on magnetic field strength and induced power

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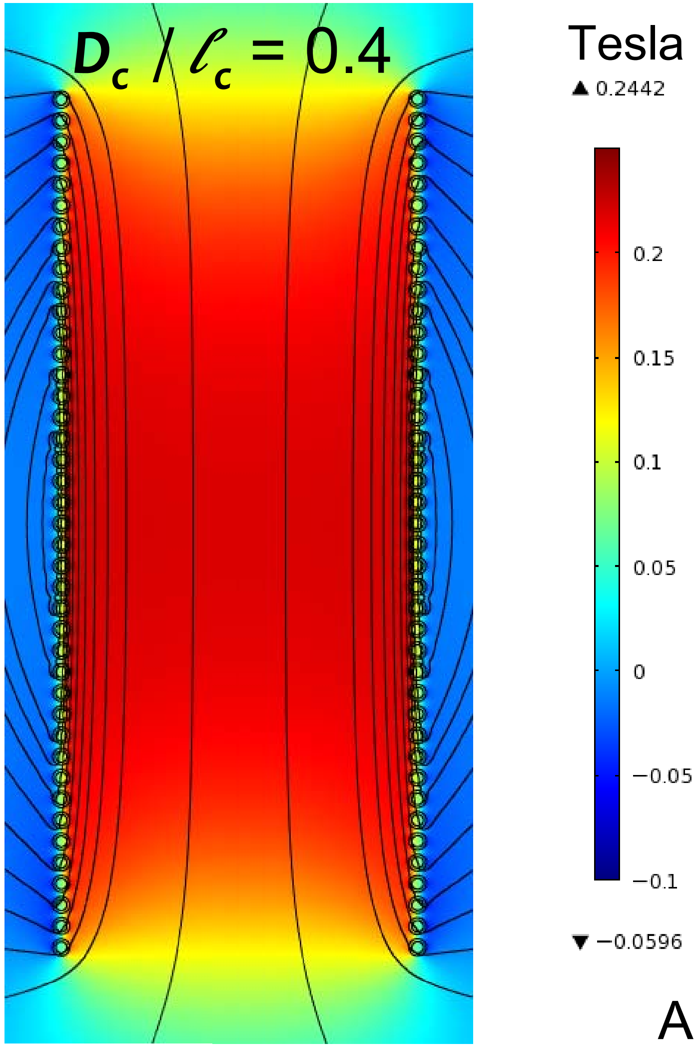
Induction Coil Theory

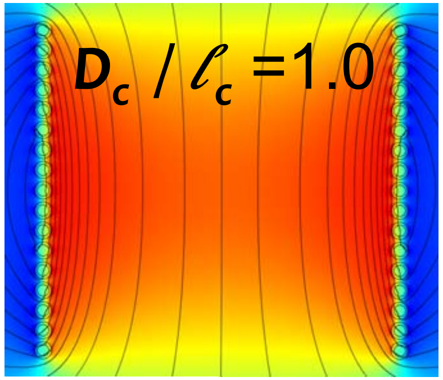
- Induced current density in the load (J = Curl H), is classically solved assuming a one dimensional system (no variation of H with coil length or radial position in the air gap), i.e. a ‘long coil assumption’: Heaviside (1892), Lord Kelvin (1890), Burch and Davis (1926), Dwight and Bagai (1935), etc.

- Short coil corrections to account for magnetic field deviation from infinite or ‘long’ coil: Burch and Davis (1928), Vaughan and Williamson (1945), Baker (1957), and Tudbury (1960).

'Long' vs. 'Short' Coil Peak Flux Density

(z-component Tesla's, 50 Hz, 1000A RMS, COMSOL®)





Infinite coil: $B_\infty = \mu_o H_\infty = \mu_o N_c I_c / \ell_c$

Real coil: $B_o = k_N B_\infty = k_N \mu_o N_c I_c / \ell_c$

k_N = Nagaoka's induction coefficient of 1909

$$k_N = 1/[1 + 0.4502\,(D_c / \ell_c)]$$

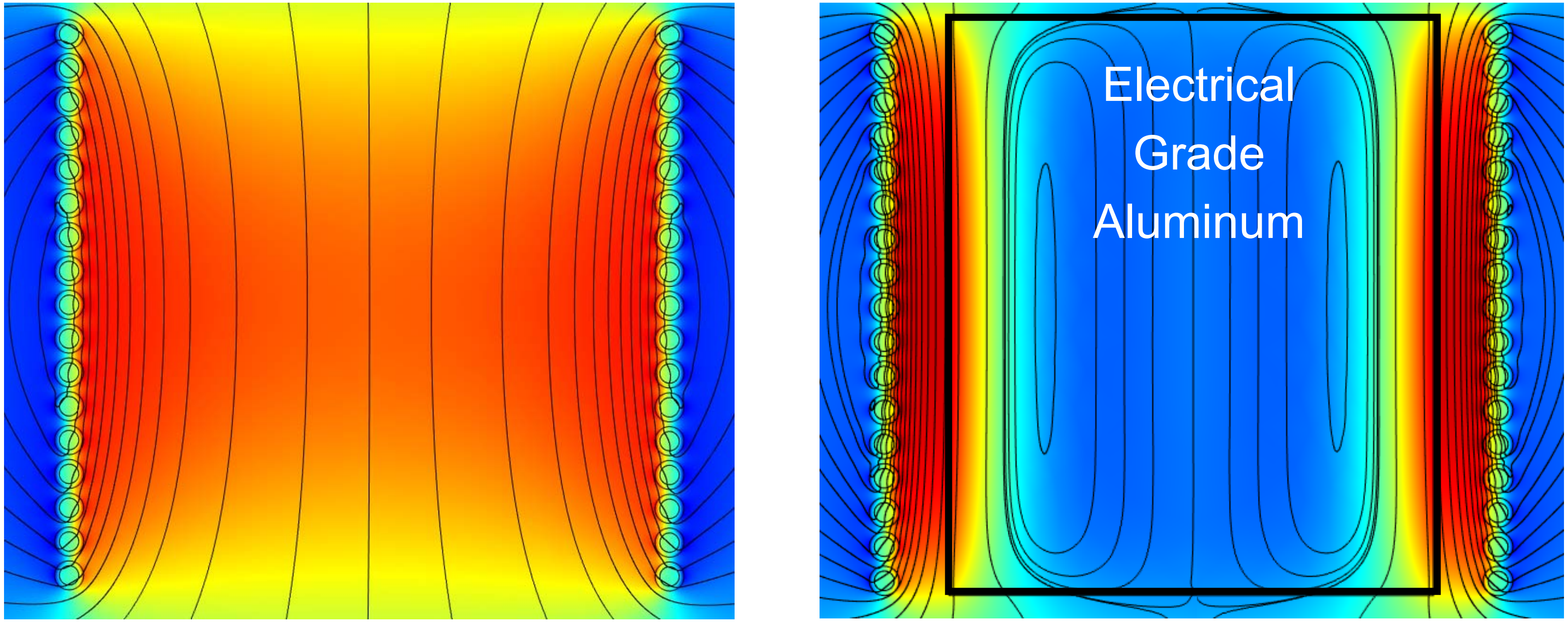
Short form empirical relation <1% error

Wheeler 1928, Knight 2010

A long coil must have $(D_c / \ell_c) < \sim 0.25$ (k_N=0.9)

Short Coil with Air Core and Work Piece

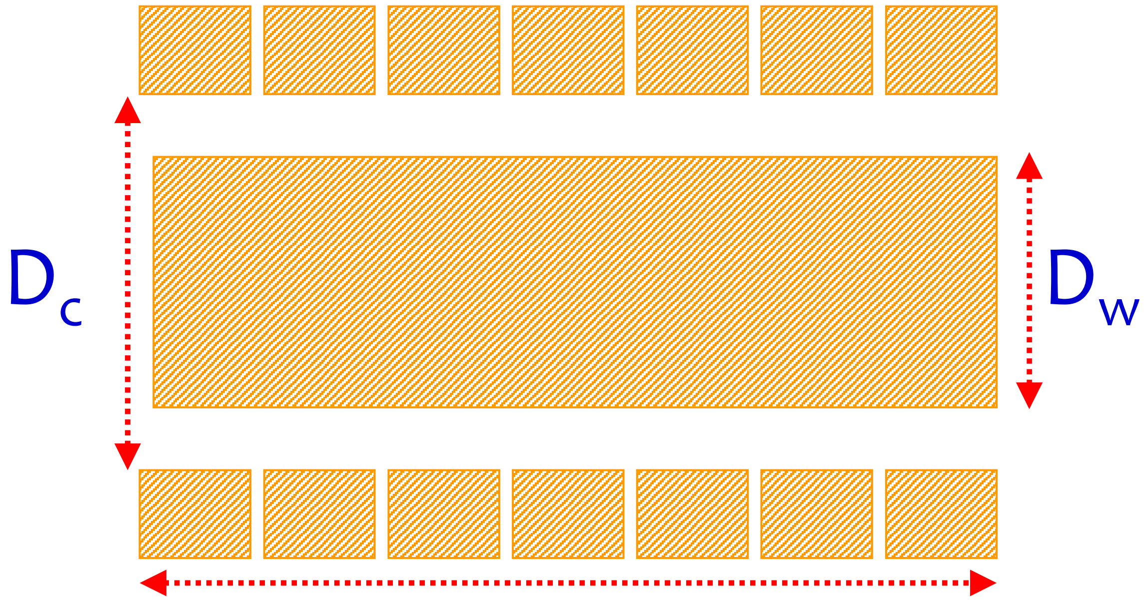
D_c / ℓ_c = 1.0, 50 Hz, 1000A RMS



The presence of the work piece alters the magnetic field in the air gap!

$$B_o^* = k_N^* B\infty = k_N^* \mu_o H_\infty = k_N^* \mu_o N_c I_c / \ell_c$$

Modified Short Coil Correction Factor



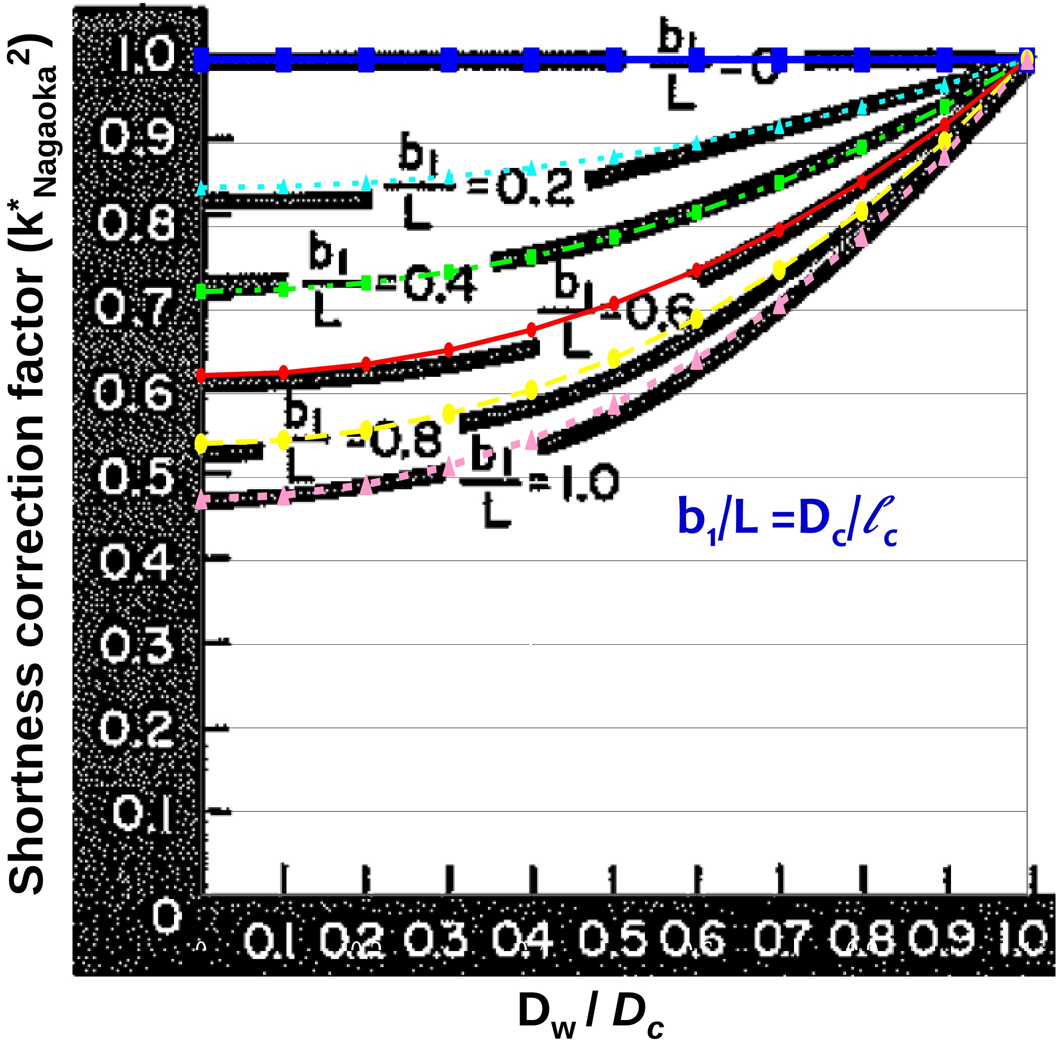
$$\ell_c = \ell_w$$

Modified Nagaoka coefficient:

Vaughan and Williamson (1945), (empirical proposal based on volume)

$$k_N^* = k_N \left(1 - [D_w / D_c]^2\right) + [D_w / D_c]^2$$

Vaughan and Williamson (1945) vs. Tudbury (1960)



Tudbury's unreferenced graph of 1960, is the square of Vaughan and Williamson's equation of 1945:

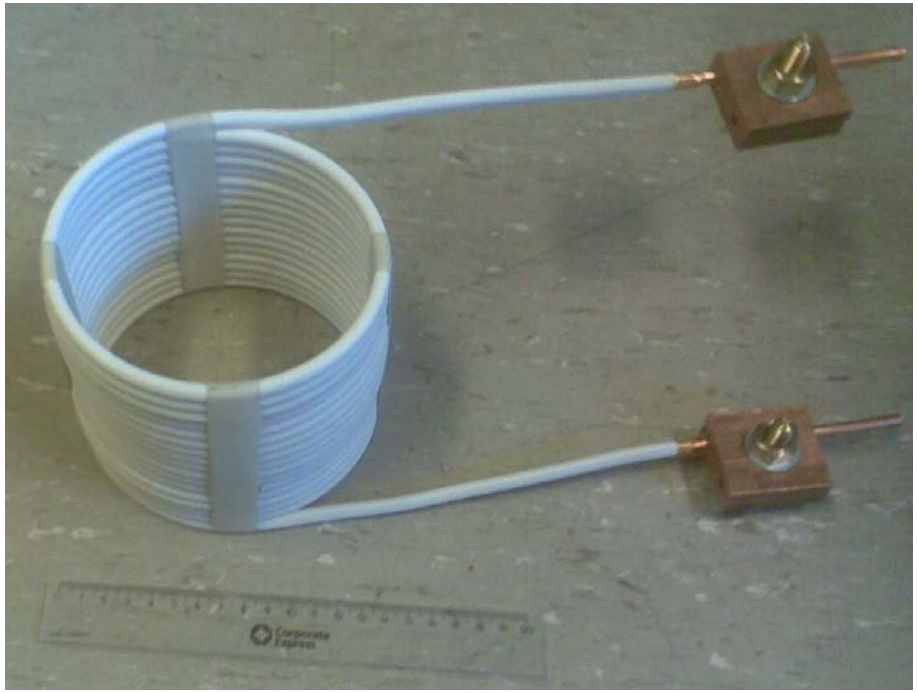
$$k_N^* = k_N (1 - [D_w / D_c]^2) + [D_w / D_c]^2$$

Air core limit:
$D_w = 0$, $D_w / D_c = 0$, $k_N^* = k_N$

Infinite coil limit:
$\ell_c = \infty$, $D_c / \ell_c = 0$, $k_N = 1 = k_N^*$

'Filled coil' limit:
$D_w = D_c$, $D_w / D_c = 1$, $k_N^* = 1$
Any coil becomes a 'long' coil

Photos of Coils 1 and 2



- 16.5 turn coil
- 111 mm long
- 126 mm diameter
- $(D_c / \ell_c) = 1.14$
- Air core short coil correction factor $k_N = 0.659$

- 41 turn coil
- 300 mm long
- 118 mm diameter $(D_c / \ell_c) = 0.39$
- Air core short coil correction factor $k_N = 0.843$

Aluminum Work Pieces and Coils Used

Work Pieces:	1	2	3	4	5
Alloy:	A356	6053	6053	6082	7108
Diameter, mm	76.8	38.6	50.5	95.3	95.4
Length, mm	192	356	1078	258	201/340
Resistivity (ohm m) at 20 C:	4.01E-08	4.11E-08	4.11E-08	3.59E-08	3.92E-08
IACS conductivity, %	43	42	42	48	44
Coil 1	1-1	1-2	1-3	1-4	1-5
Coil 2			2-3	2-4	2-5

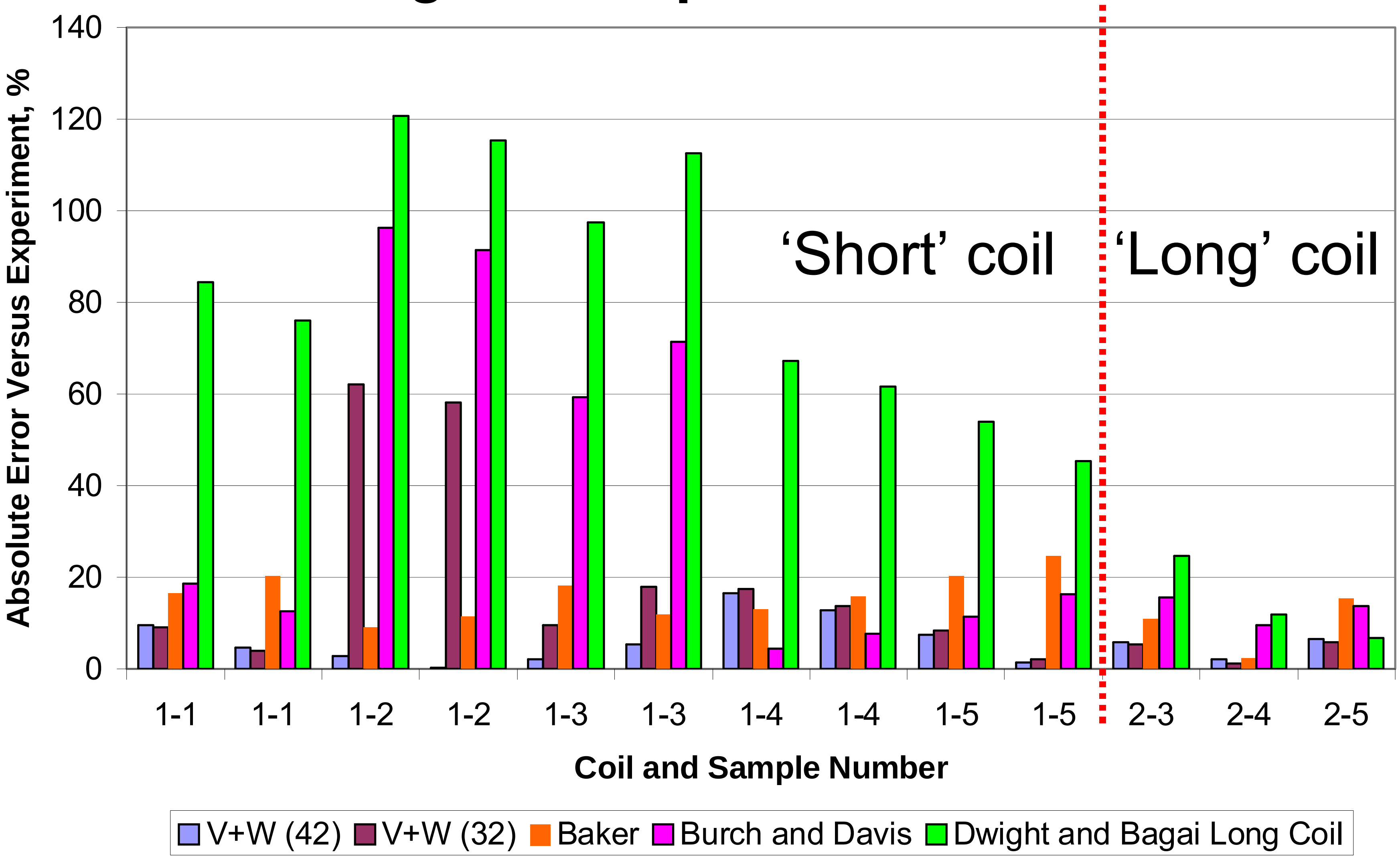
Comparison of Measured and Calculated Work Piece Power

Load Number	Measured Power, W	Equation (32)	Equation (42)	Baker Short Coil	Burch and Davis	Dwight and Bagai
1-1	692	754	759	577	821	1275
1-1	899	936	942	716	1013	1583
1-2	138	224	142	125	271	305
1-2	177	281	178	157	340	382
1-3	325	356	318	266	517	641
1-3	377	445	398	333	647	802
1-4	990	1165	1155	862	945	1656
1-4	1264	1438	1426	1064	1167	2045
1-5	1086	1176	1168	866	962	1673
1-5	1422	1454	1443	1070	1188	2067
2-3	640	674	602	569	740	798
2-4	1553	1533	1520	1591	1402	1738
2-5	1923	1810	1797	1627	1657	2054
Error	+/-~2%	16.6	6.0	14.6	33.0	67.6

Equation (32): $P_w = k_N^{*2}\, \pi\, (I_c N_c)^2 \rho_w (D_w - \delta_w) / (\delta_w\, \ell_c)$

Equation (42): $P_w = k_N^{*2} \sqrt{2}\, \pi\, (I_c N_c)^2 \rho_w\, \xi_w\, \varphi(\xi_w) / \ell_c$

Comparison of Different Calculation Methods Against Experimental Data



Conclusions

- Tudbury's unreferenced graph of 1960 is the square of Vaughan and Williamson's modified short coil correction factor of 1945.

$$k_N^* = k_N (1 - [D_w / D_c]^2) + [D_w / D_c]^2$$

- Vaughan and Williamson's equation is considered valid for high conductivity metal (over 40% IACS/23.2 MS/m), from line frequency and above.

- Equation (42), incorporating Vaughan and Williamson's equation, appears to be the most accurate of the classical tools available, with typical errors of 6% in the estimation of work piece power for any combination of load and coil geometry, at 50 Hz using aluminum work pieces.

$$P_w = k_N^{*2} \sqrt{2}\, \pi\, (I_c N_c)^2 \rho_w \xi_w \varphi(\xi_w) / \ell_c \qquad (42)$$

- Long coil formulae (e.g. Dwight and Bagai), can only be used for very long coils *$(D_c / \ell_c) < \sim 0.25$.* Large errors, e.g. >70% will result in the estimation of work piece power for typical induction coils dimensions.

Future work

- High Accuracy "Hall" probe Gauss meter measurements of coil magnetic field strengths inside and outside, "aircore" and loaded coils to experimentally verify magnetic field predictions of analytical and finite element models.

- A journal paper with improved Vaughan and Williamson formula covering a frequency range from 50 Hz - 500 kHz (2011).

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Thank you for your attention !

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Short Coil Peak Magnetic Flux Density –

Z Component, 50 Hz, 1000A RMS